What is the best way for me to get started?

About a half hour of preparation will get your kit ready for the year.

- Punch out the Math Maze cards from the perforated sheets. Each sheet and the individual cards are labeled by week number and activity number. To keep the cards organized in your kit, you may want to bundle together each deck of cards with a rubber band or store them in plastic resealable bags.

- Punch out the digit cards, shape cards, and the colored squares. Put the punchouts in labeled bags and set them aside.

- Cut out the protractors along the dotted lines and store them in a bag.

- The kit comes with a bag of un-labeled number cubes. Use a fine-point permanent marker to label them with the digits 1–6. If you need to change a cube’s labels, use small, round stickers.

- Remove the Math Jumble activity poster from the kit. Slit the poster along the cut lines indicated on the poster. Insert paper clips and tape the paper clips to the back of the poster. When you do the Math Jumble activities, slide the digit cards or shape cards under the paper clips to attach them to the poster.

To familiarize yourself with the four different types of activities, you may want to read the first week’s activities in the Instructor’s Guide. Then look over the rest of the materials in the kit.

You may decide to start with Week 1 Activity 1 and work through the activities in order, or give the students some or all of the diagnostic tests to determine their strengths and weaknesses. The diagnostic tests are located on pages 181–190 of the student book. There are five separate two-page tests for the five different mathematics strands — number, basic operations, geometry, measurement and data collection, and algebra and patterns. Answers to the tests are found in the Instructor’s Guide, beginning on page 213.

The test items are correlated to the 180 activities in the program. You may decide to pick and choose activities based on specific math strands or concepts.
Materials
Student page 1
Math Maze Cards (Week 1 Activity 1)

Concepts and Handbook References
Multiply prime numbers. (MOC 058, 143–148)
Use exponents. (MOC 071)
Find prime factors. (MOC 056, 061)

Background
In the expression $3^2$, the three is the base and the two is the exponent. The exponent tells how many times to use the base as a factor.

$$3^2 = 3 \times 3 = 9 \quad 5^3 = 5 \times 5 \times 5 = 125$$

The expression $3^2$ is read three squared or three to the second power.

The expression $5^3$ is read five cubed or five to the third power.

Get Started
Discuss the meaning of exponents, and then ask the following questions.

• How would you write $4 \times 4 \times 4$ using exponents? ($4^3$)

• Which is greater, $3^2$ or $2^3$, or are they equal? ($3^2 > 2^3$ since $9 > 8$)

• How would you find the value of $2 \times 3^2 \times 5$? ($2 \times 3 \times 3 \times 5 = 90$; students should notice that they can multiply $2 \times 5$ first in order to find the product with mental math)

Today's Challenge
Distribute the 18 Math Maze cards for week 1. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, "Who has the card with the answer to the question just read?" Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student's card.

The correct sequence of questions and answers is shown on page 181.

Student page 1 When the group has finished playing the game, have students open their books and complete the Today's Challenge activity on page 1 in the student book.

Answers for student page 1: 1. $2 \times 3 \times 5 \times 7$; 210
2. $3^2$; 27 3. $2^3 \times 7$; 196 4. $2 \times 5^2$; 250 5. $3 \times 5^2$; 75 6. $2^3 \times 5$; 40 7. $2 \times 5^2 \times 7$; 350 8. $2^3 \times 7$; 56 9. $3^2 \times 7$; 63 10. $3^2 \times 2$; 54

Go Further
Student page 1 Have students complete this section of the student page.

Answers for student page 1: 11. $2^3 \times 3$ 12. $3 \times 7^2$
13. $2^3 \times 3 \times 5^2$

Assessment
Student self-assessment page 1 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students notice that numbers with zero in the ones place will always have two and five among their factors?
Materials
Student page 2
Number cubes (1–6)
Chart paper and markers

Concepts and Handbook References
Collect and organize data in a table. (MOC 285)
Analyze fairness of a game. (MOC 465, 466)

Get Started
Go over the game rules at the top of student page 2. Ask students whether they think the game is fair (either player is equally likely to win). Discuss how to gather data from other pairs without disturbing the group, then have the students get into pairs and play the game 50 times.

Today’s Challenge
Student page 2 Pass out two number cubes to each pair of students. You may want to pre-assign a record keeper for each pair.

Answers for student page 2: 1–2. Check students’ work. 3. This is not a fair game, and results for Player A should be better than for Player B.

Look Ahead
Draw this table on chart paper. You will use it as a sample space next time.
Materials
Student page 3

Concepts and Handbook References
Create a sample space. (MOC 463)
Use absolute value. (MOC 050)
Find probability. (MOC 465)
Create a fair game. (MOC 466)

Background
A sample space shows all possible outcomes of a game or experiment. In today’s game, Player A tossing a six and Player B tossing a one is a different outcome than Player A tossing a one and Player B tossing a six.

Absolute value is the distance of a number from zero on a number line. Direction from zero is ignored with absolute value.

Example: ‖−4‖ = 4 → The absolute value of negative four is four.

Go Further
Student page 3 Have the students fill in the sample space so that they can use it to find the probability of rolling a difference from zero through five with two number cubes. As students fill in their tables, you should fill in your large copy of the chart and display it so that students can check their results. Have them work with their partners to complete the page.

Answers for student page 3: Check students’ work.
Possible answers are provided.

Things we discovered:
• A difference of zero occurs six times.
• A difference of one occurs ten times.
• A difference of two occurs eight times.
• A difference of three occurs six times.
• A difference of four occurs four times.
• A difference of five occurs two times.
• There are more ways to get zero, one, and two than to get three, four, and five.

2. A. 6/36 B. 10/36 C. 8/36 D. 6/36 E. 4/36 F. 2/36 3. One way to make the game fair is to let Player A win if the difference is zero, one, or five and let Player B win if the difference is two, three, or four.

Assessment
Student self-assessment page 3 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students know how to create a sample space to illustrate and analyze outcomes from an event? Do students recognize when a game is not fair?
Materials
Student page 4
Blank paper (heavyweight if possible) or index cards

Concept and Handbook References
Determine the length of the base or height of a triangle when the area is given and either the base or height is known. (MOC 356, 561)

Get Started
Review why the formula for the area of a triangle works by drawing a rectangle with one diagonal. (Figure 1) The area of the rectangle is the length of its base times its height and the triangle’s area is one-half of the rectangle’s area. The formula can also be demonstrated with a non-right triangle by enclosing it in a rectangle. (Figure 2)

\[ \text{Triangle Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh \]

Student page 4 Have students complete the two examples at the top of student page 4. In Example 1, the area is 12 square meters and the base is 6 meters long. Have students explain how the height can be found and apply that explanation to example 2. (Substitute known values into formula and simplify.)

For example 1, \[ A = \frac{1}{2}bh \]
\[ 12 = \frac{1}{2}(6)h \]
\[ 12 = 3h; h = 4 \text{ meters} \]

For example 2, \[ A = \frac{1}{2}bh \]
\[ 12 = \frac{1}{2}(8)h \]
\[ 12 = 4h; h = 3 \text{ meters} \]

Today’s Challenge
Student page 4 Have students complete the table. Explain that each row has two triangles that have the same area but different bases and heights. In each case, students are to find the missing length.

Answers for student page 4: 1. 2 ft  2. 6 ft  3. 8 cm  4. 6 cm  5. 4 yd  6. 2 yd  7. 2 in.  8. 1 in.  9. 5 yd  10. 10 yd  11. 8 ft  12. 10 ft

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 12 small pieces of paper or 12 index cards. Have the students use one slip of paper or card to copy the completed diagrams from the right side of each box in the table on student page 4, including the units of measure. Students will not copy the information about the area or the two examples at the top of the page.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a 3 \( \times \) 4 array. The first player turns over any two cards. If the cards match (have the same area), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 4 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students find a missing length in a triangle when given the area and the length of a base or height?
Materials
Student page 5

Concepts and Handbook References
Use variables to write an equation. (MOC 205)
Analyze answer choices with special attention to implied units. (MOC 529)

Get Started
Write this problem on the board.

I bought two adult tickets and some children’s tickets to the movie for a price of $37.50. If adult tickets cost $12.00 each and children’s tickets cost $4.50 each, how many children’s tickets (t) did I buy?

Write this list of answer choices and tell the class you want them to select an equation that could be used to help them solve the above problem.

A. 37.50 = 12(2) + 450t
B. 3750 = 1200 + 450t
C. 3750 = 1200(2) + 4.50t
D. 37.50 = 12(2) + 4.50t

In the discussion, be sure to cover these points.
• The equations in A and C mix dollars and cents as if they were the same.
• The equation in B uses only cents, but it does not account for two adult tickets.
• Equation D is in dollars and accounts for the correct number of adults. The answer is D.

Student page 5 Work with the class to analyze the equations in the Get Started section. (B is correct. The equation in A mixes dollars and cents, equation C is in cents but the cost is supposed to be in dollars, and equation D is incorrect because it adds the minimum charge fee to the cost per hour before multiplying by the number of hours.)

Today’s Challenge
Student page 5 Have students work independently on student page 5. Encourage them to carefully analyze the answer choices.


Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 5. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 5 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students consider the units when analyzing an equation?
Materials
Student page 6
Math Maze Cards (Week 2 Activity 6)

Concept and Handbook Reference
Use correct mathematical terminology. (MOC 572)

Background
Often students are tripped up on standardized tests not by what they do not know mathematically but by the terminology. They may know, for example, how to divide, but not what a quotient is. Encourage students to use proper vocabulary when doing oral practice.

Get Started
Ask students to list all the words they can think of that have to do with algebraic equations, expressions, properties, and fractions. Make a master list with examples and keep it where students can see it.

You may wish to begin a class math dictionary with a page for each letter. Enter new math terms as they come up. Alternatively, there are some good commercial math dictionaries on the market that will be helpful to have. Great Source’s Math On Call handbook has a useful glossary.

Today's Challenge
Distribute the 18 Math Maze cards for week 2. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 182.

Student page 6 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 6 in the student book.

Answers for student page 6: 1. variable, solution 2. integers, whole numbers, counting numbers 3. distributive, commutative, associative 4. reciprocals, opposites, greatest common factor, least common multiple

Go Further
Student page 6 Have students refer to the class list if they need ideas for completing exercise 5.

Answers for student page 6: 5. Check students’ work. Possible answers include real number, inverse operation, identity element, term

Assessment
Student self-assessment page 6 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students consistently use the proper terms when discussing mathematical topics?
Materials
Student page 7
Rulers
Isometric dot paper (page 218)

Concepts and Handbook References
Use the Look for a Pattern and Draw a Diagram problem-solving strategies. (MOC 484, 483)
Work with reflections. (MOC 388)

Background
Students will examine a pattern made with equilateral triangles and rhombi (plural of rhombus) whose side length is the same as the triangle's side length. Although the pattern only involves two polygons, it is an ABCD pattern, since the orientation of the polygons is also an important aspect of the pattern.

![Diagram of pattern]

These are reflections of each other as are figures A and C.

Get Started
Draw this diagram on the board and discuss the pattern it makes.

Look Ahead
Tell students that next time they will devise a way to predict the final polygon in the pattern for any n without drawing a diagram.

Ask students to predict the figure in the fifteenth position (rhombus). Draw diagrams if needed. Discuss the fact that this is an ABC pattern even though there are only two polygons, because triangles that are reflections of each other count as different figures in this pattern.

Today's Challenge
Student page 7 Have the students work by themselves to draw Figures 7–12. After they have completed their drawings, have them compare their drawings with a friend.

Answers for student page 7:

1. \( n = 7 \)
   ![Diagram of n = 7]

2. \( n = 8 \)
   ![Diagram of n = 8]

3. \( n = 9 \)
   ![Diagram of n = 9]

4. \( n = 10 \)
   ![Diagram of n = 10]

5. \( n = 11 \)
   ![Diagram of n = 11]

6. \( n = 12 \)
   ![Diagram of n = 12]
Materials
Student page 8
Tracing paper (optional)

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484, 483)
Interpret remainders. (MOC 182)

Get Started
Ask student volunteers to display their diagrams from activity 7. Discuss any disagreements.

Go Further
Student page 8 Have students work in pairs to analyze the data they collected last time. If students struggle with exercises 3–5, suggest that they trace copies of the diagrams, then physically test whether they join smoothly to create the new figure.

Answers for student page 8: Check students’ work. Possible answers are provided.

Things we discovered:
• The polygon on the end of figures in which n is a multiple of four is a rhombus leaning to the right.
• The polygon on the end of figures in which n is one more than a multiple of four is a triangle pointing up.
• The polygon on the end of figures in which n is two more than a multiple of four is a rhombus leaning left.
• The polygon on the end of figures in which n is three more than a multiple of four is a triangle pointing down.

1. A) triangle pointing down B) rhombus leaning right C) triangle pointing up D) rhombus leaning left 2. One way is to divide n by four. If there is no remainder, the final polygon is a rhombus leaning right. If R = 1, it is a triangle pointing up. If R = 2, it is a rhombus leaning left. If R = 3, it is a triangle pointing down. 3. Yes; figures for n = 4 and n = 8 both end with a rhombus leaning right. 4. No; there is a gap in the middle if you tack the figure for n = 5 to itself. 5. Yes; figures for n = 6 and n = 10 both end with a rhombus leaning left and there is no gap where n = 4 and n = 6 meet because figure n = 4 is a complete pattern and the beginning of figure n = 6 is the beginning of the pattern repeat.

Assessment
Student self-assessment page 8 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students make predictions based on patterns?
Materials
Student page 9
Math Jumble activity poster and digit cards

Concept and Handbook Reference
Use divisibility rules for two, three, six, and nine.
(MOC 069)

Background
• A number is divisible by two if the last digit is even.
• A number is divisible by three if the sum of its digits is divisible by three.
• A number is divisible by six if it is divisible by two and by three.
• A number is divisible by nine if the sum of its digits is divisible by nine.

Get Started
Begin by reviewing the divisibility tests for two, three, six, and nine. Ask students to name a three-digit number that is divisible by two. Make a list of at least six numbers. Now use the divisibility rules to check the numbers for divisibility by three. Finally, ask students for a three-digit number that is divisible by nine but not six. This means that it will be an odd number whose digits have a sum divisible by nine.

Today's Challenge
Use the 0-9 digit cards to construct this poster.

| 7 | 1 | 3 | 4 |
| 1 | 5 | 6 | 7 |
| 3 | 2 | 3 | 2 |
| 6 | 9 | 8 | 0 |

Explain that the object of today's Math Jumble is to find three-digit numbers that are divisible by two, three, six, or nine.

A three-digit number can be made with any three adjoining digits (horizontally, vertically, and/or diagonally) on the poster. Digits on the poster can be used more than once, but not in the same number. For example, the one and three in the center of the top row and the five in the second row can be used to form the numbers 135, 531, and 315, all of which are divisible by three and nine but not by two or six. Record the numbers that students make for each problem.

• Possible numbers divisible by two: 364, 720, 932
• Possible numbers divisible by three: 363, 720, 936
• Possible numbers divisible by six: 162, 720, 936
• Possible numbers divisible by nine: 162, 720, 936

Student page 9 Have students use the Math Jumble on student page 9 to find numbers divisible by the given number.

Answers for student page 9: Check students' work. Possible answers are provided. 1. 298, 764, 902, 816 2. 630, 636, 816, 987 3. 816, 102, 720, 360 4. 360, 720, 981, 189 5. 987, 603, 627, 813 6. 298, 764, 902, 892

Go Further
Student page 9 Have students use the grid on the student page to create a Math Jumble to share with a friend.

Answers for student page 9: 7–9. Check students' work. Grids and expressions will vary.

Assessment
Student self-assessment page 9 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students determine whether a number is divisible by two, three, six, or nine? Can they find numbers that are divisible by one number but not by another?
Materials
Student page 10
Calculators
Chart or overhead transparency of blank fill-in grid
(save for future activities)

Concepts and Handbook References
Use proportions to solve percent problems.
(MOC 444, 446–449)
Use variables. (MOC 202–203)
Show percents in machine-scoreable form.
(MOC 532)

Background
A percent can always be written as a ratio: \( x\% = \frac{x}{100} \).

Example: What percent is 32 of 46?
\[
\frac{x}{100} = \frac{32}{46}
\]
\[
46x = 3200
\]
\[
x = 3200 \div 46
\]
\[
\approx 70
\]

Since the problem was set up in terms of hundredths, think of the solution as \( x\% \approx 70\% \).

To find percent increase, find the amount of increase, then find what percent that number is of the original amount.

Example: The elephant weighed 500 pounds at birth and now weighs 620 pounds. By what percent has its weight increased?

Find amount of increase. 620 – 500 = 120
Find percent of increase. \[
\frac{x}{100} = \frac{120}{500}
\]
\[
500x = 12,000
\]
\[
x = 12,000 \div 500
\]
\[
= 24
\]
Answer the question. The elephant’s weight increased 24%.

Get Started
Review finding percent using a proportion, then review percent increase.

Student page 10 Point out that the grids in a grid-
ded response form may not have a percent symbol.
Discuss ways to show a percent without using the
symbol, then read and discuss the problem at the
top of student page 10. (B and D are correct, as the
machine does not require answers to be flush left or
to have a zero in the ones place when the number
is less than one. A uses an illegal symbol; C is not
equivalent to 14%.)

Today’s Challenge
Student page 10 Encourage students to carefully
mark the grids for exercises 1–4.

Answers for student page 10:

1. 12. 2. 23. 7. 3. 0.9. 3. 4. 0.7. 4.

5. Check students’ advice. They might mention the
need to account for the fact that the response grid
has no percent symbol and the need to read test
directions carefully.

Go Further
On a separate sheet of paper, have each student
write his or her name and create two problems sim-
lar to those on student page 10. They should write
the correct answers on the back. Have them share
their problems with a friend, who will solve the
problems, then sign his or her name. If there is a
disagreement, students should edit their work.

Assessment
Student self-assessment page 10 Have students cir-
cle one of the two choices to describe how they feel
about the activity.

Assessment tip Do students confidently and accu-
rrately compute percent increase?
Materials
Student page 11
Math Maze Cards (Week 3 Activity 11)

Concept and Handbook References
Review conversion equivalents in the U.S. customary
and metric measurement systems. (MOC 535, 536)

Background
Decimal numbers should be read very carefully.
1. Read the whole-number part if it is greater
   than zero.
2. Say and instead of point.
3. Read the part to the right of the decimal point
   as a fraction whose denominator is the name of
   the farthest-to-the-right place.

Examples:

<table>
<thead>
<tr>
<th>See</th>
<th>Say</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>three and five tenths</td>
</tr>
<tr>
<td>0.124</td>
<td>one hundred twenty-four thousandths</td>
</tr>
<tr>
<td>100.024</td>
<td>one hundred and twenty-four thousandths</td>
</tr>
</tbody>
</table>

Get Started
Ask about things we measure (time, temperature,
capacity, distance, weight) and some tools we use
for measuring (measuring spoons and cups, rulers,
clocks, scales, thermometers, etc.) Then, ask about
measurement equivalents.

- How many cups are in a quart? (4); in 3.5 quarts? (14)
- How many meters are in a kilometer? (1000); in
  1.5 kilometers? (1500)
- How many ounces are in a pound? (16); in 1.5
  pounds? (24)
- How many inches are in a yard? (36); in \( \frac{2}{3} \) yard? (24)

Today's Challenge
Distribute the 18 Math Maze cards for week 3. Each
student should receive at least one card, but since
all cards need to be distributed, some students may
need to get more than one card. Use the cards to
play the Math Maze game.

Instructions for playing Math Maze Ask students to
look at their cards. Ask one student to read the
question that is written on his or her card. Next ask,
"Who has the card with the answer to the question
just read?" Ask that student to read the answer, and
then read the question on his or her card. Play con-
tinues until all questions have been answered. The
last answer to be read should be the answer on the
first student's card.

The correct sequence of questions and answers is
shown on page 183.

Student page 11 When the group has finished play-
ing the game, have students open their books and
complete the Today's Challenge activity on page 11
in the student book.

Answers for student page 11: 1. 5280 2. 30 3. 25
4. 45 5. 42 6. 16 7. 24 8. 0 9. 2500 10. 10

Go Further
Student page 11 Encourage students to write stories
or poems for exercise 11.

Answers for student page 11: 11. Check students’
work.

Assessment
Student self-assessment page 11 Have students cir-
cle one of the two choices to describe how they feel
about this activity.

Assessment tip Can students identify common U.S.
customary and metric units and equivalent units
within each system?
Data Collect It

Week 3 • Activity 12

Today's Challenge
Student page 12 Have students work in pairs to find the perimeters and areas of Figures 5–10. Have them record their findings in the tables, then check their work with another pair of students.

Answers for student page 12:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter (in units)</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>12</td>
<td>14</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

2.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (in triangular units)</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>12</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>

Look Ahead
Prepare a chart showing the answers for this page. This will help students be sure they are working with accurate data next time.

Materials
Student page 12
Isometric dot paper (page 218, optional)
Chart paper and markers

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)

Get Started
Draw this diagram on the board and tell the students they will be studying the same pattern they used on student pages 7–8 to predict perimeter and area.

\[ n = 1 \]
\[ \begin{align*}
1 \text{ unit} & \quad 1 \text{ unit} \\
1 \text{ unit} & \quad 1 \text{ unit}
\end{align*} \]

\[ n = 2 \]
\[ \begin{align*}
1 \text{ unit} & \quad 1 \text{ unit} \\
1 \text{ unit} & \quad 1 \text{ unit}
\end{align*} \]

\[ n = 3 \]
\[ \begin{align*}
1 \text{ unit} & \quad 1 \text{ unit} \\
1 \text{ unit} & \quad 1 \text{ unit}
\end{align*} \]

\[ n = 4 \]
\[ \begin{align*}
1 \text{ unit} & \quad 1 \text{ unit} \\
1 \text{ unit} & \quad 1 \text{ unit}
\end{align*} \]

Work with the class to find and record the perimeter and area of Figures 1–4. Perimeter is in one side-length units and the area is in one-triangle units.
**Materials**
Student page 13
Isometric dot paper (page 218, optional)

**Concepts and Handbook References**
Use the Look for a Pattern problem-solving strategy.  
(MOC 484)
Write equations to describe functional relationships.  
(MOC 205, 244)

**Get Started**
Double check to be sure students have their work for page 12. Display the chart you made last time so that they can check that the data in their tables are correct.

**Go Further**
Student page 13 Have students work in pairs to analyze their data from student page 12. Have them try to predict the perimeter and area of Figures 26 and 43 without drawing the diagrams.

**Answers for student page 13:** Check students’ work. Possible answers are provided.

**Things we discovered:**
- The perimeters of consecutive Figures go up two, then one, then two, then one, etc.
- The areas of consecutive Figures go up two, then one, then two, then one, etc.
- The perimeters of consecutive odd-numbered Figures form the sequence 3, 6, 9, 12, 15, and so on.
- The perimeters of consecutive even-numbered Figures form the sequence 5, 8, 11, 14, 17, and so on.
- Both even- and odd-Figure sequences of both perimeters and areas differ from one term to the next by three.
- The areas of consecutive odd-numbered Figures form the sequence 1, 4, 7, 10, 13, and so on.
- The areas of consecutive even-numbered Figures form the sequence 3, 6, 9, and so on.

1. 13, 22; If \( n \) is even, there are \( \frac{n+1}{2} \) triangles. If \( n \) is odd, there are \( \frac{n+1}{2} \) triangles. 2. 13, 21; if \( n \) is even, there are \( \frac{n}{2} \) rhombi. If \( n \) is odd, there are \( \frac{n-1}{2} \) rhombi. 3. 41 units, 66 units; the ends always account for two perimeter units. If \( n \) is even, the triangles account for \( \frac{n}{2} \) additional units and the rhombi account for \( n \) additional units so, for even \( n \), \( p = \frac{n}{2} + n + 2 \) or \( \frac{3n}{2} + 2 \). If \( n \) is odd, triangles account for \( \frac{n-1}{2} \) additional units and rhombi account for \( \frac{n-1}{2} \) additional units so, for odd \( n \), \( p = \frac{n-1}{2} + (n - 1) + 2 \) or \( \frac{3(n-1)}{2} \). 4. 39 triangular units, 64 triangular units; if \( n \) is even, there are \( \frac{n}{2} \) triangles and rhombi, so for even \( n \), \( A = \frac{n}{2} + n \) or \( \frac{3n}{2} \). If \( n \) is odd, there are \( \frac{n+1}{2} \) triangles and \( \frac{n-1}{2} \) rhombi, so for odd \( n \), \( A = \frac{n+1}{2} + n - 1 \) or \( \frac{3n-1}{2} \).

**Assessment**
Student self-assessment page 13 Have students circle one of the two choices to describe how they feel about this activity.

**Assessment tip** Can students find patterns within patterns?
Materials
Student page 14
Slips of paper containing multiples of six

Concept and Handbook References
Identify number concepts such as multiples, factors, and exponents. (MOC 056, 067, 071)

Get Started
Write the multiples of six (6 through 72) in a vertical list on the board. Ask students what patterns they see. Probe for these points.
• All of the numbers are even.
• The digits in the ones place repeat 6, 2, 8, 4, 0.
• The digits in the tens place repeat except when there is a four in the ones place. (12, 18, 24, 30, 36, 42, 48)
• All of the numbers are multiples of both two and three.
• The sum of the digits in the tens and ones places is divisible by three.

Today’s Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with one of the multiples of six (6 through 72) written on it. You do not need to use all of the multiples of six, but be sure that one student receives the number 36, since that number will be the “Fantastic Finalist.”

Have all students hold their numbers and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
• If your number has a one in the tens place, sit down. (12, 18)
• If your number is greater than or equal to 24, sit down. (66, 72)

• If the sum of the digits in your two-digit number is even, sit down. (24, 42, 48, 60)
• If your number is less than the number of ounces in two pounds, sit down. (6, 30)
• If your number is divisible by 27, sit down. (54)

At this point, only the student holding the number 36 should still be standing. That student is the “Fantastic Finalist.”

Go Further
Student page 14 Have students complete the activity on the student page.

Answers for student page 14:
1-5.

64 44 32 19

48 72 46 12

54 84 20 60

6. 32 7. Check students’ work. Possible answers: 32 is a power of two. 32 cents can be made with a minimum of four coins. There are fewer than 32 inches in a yard.

Assessment
Student self-assessment page 14 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students identify factors and multiples of a number? Can they raise a number to a power?
Materials
Student page 15

Concepts and Handbook References
Practice spatial visualization through reasoning by analogy. (MOC 487)
Rotate or reflect figures. (MOC 386, 388)

Get Started
The ability to use analogous reasoning involves recognizing similarities and differences. Questions involving analogous reasoning are often used on intelligence tests and on college entrance exams. Ask the class what word they might use to complete the sentence, Night is to day as black is to ______. (white) Explain that what they just solved is called an analogy. Discuss the fact that students are often asked to reason by analogy. Draw this diagram on the board.

\[
\begin{align*}
\text{Circle} & : \quad \text{Shaded semicircle} = \\
& : \quad \text{Square} : \quad \text{Shaded half-square}
\end{align*}
\]

Discuss the fact that the analogy is based on two relationships, cutting the figure in half and shading the figure in.

Student page 15 Do the Get Started activity at the top of student page 15 as a class activity. Thoroughly analyze and discuss each analogy. (A is correct. D is correct if you see the relationship as add one square to width. D is incorrect if you see the relationship as double the width. In B, cone is the wrong three-dimensional figure, it should be sphere. For C, since d follows c in the alphabet, h should follow g or i should follow h to correct the analogy.

Today's Challenge
Student page 15 Encourage students to think carefully about similarities and differences among the parts of these analogies. Some students may need to write these characteristics in order to be sure they have correctly completed the analogies.

Answers for student page 15:
1. pyramid
2.  
3.  
4.  
5. Check students’ work. Possible answers:

6. Check students’ advice. It should involve looking for similarities and differences which might include shading, reflections, rotations, or going from two dimensions to three dimensions.

Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 15. They should write or draw the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement that turns out not to reflect a true alternate solution, students should edit their work.

Assessment
Student self-assessment page 15 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip When solving a problem using analogous reasoning, do your students ask themselves, What is the same about the figures? What is different about the figures?
Materials
Student page 16
Math Maze Cards (Week 4 Activity 16)

Concepts and Handbook References
Solve one-step equations. (MOC 241)
Compute with fractions and whole numbers. (MOC 086)
Use inverse operations. (MOC 100, 130, 156, 183)

Background
A variable is any word, phrase, or symbol that represents a number in a mathematical expression. In three times my number is 12, the variable is my number. Other ways to write the same expression are:

\[
3x = 12 \\
3 \times \underline{\text{______}} = 12 \\
3n = 12
\]

Get Started
Talk with students about variables, and then do a few practice examples.

- \(3n = 42\). What is \(n\)? (14)
- \(x - 23 = 18\). What is \(x\)? (41)
- \(n \div 3 = 4\). What is \(n\)? (12)
- \(\frac{3}{4}n = 15\). What is \(n\)? (20)

Often what helps students to solve these problems is to work backward. This involves using inverse operations: multiplication and division are inverse operations; so are addition and subtraction. For example, if three times \(n = 42\), then \(n = 42 \text{ divided by} 3\) because division undoes multiplication. If \(\frac{3}{4}\) of a number is 15, then \(\frac{3}{4}\) times that number is 15 and 15 divided by \(\frac{3}{4}\) is the number.

Talk with students about the difference between these two equations.

\[
n - 8 = 5 \\
8 - n = 5
\]

The first equation is solved by addition, the inverse of subtraction \((5 + 8 = n)\) while the second may be solved by writing the related subtraction problem \((8 - 5 = n)\).

Today’s Challenge
Distribute the 18 Math Maze cards for week 4. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 184.

Student page 16 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 16 in the student book.

Answers for student page 16:
1. \(n - 16 = 18\); \(n = 34\) 2. \(9n = 108\); \(n = 12\)
3. \(4n = 144\); \(n = 36\) 4. \(n + 32 = 58\); \(n = 26\)
5. \(\frac{1}{4}n = 21\); \(n = 63\) 6. \(n^2 = 64\); \(n = 8\) or \(n = -8\)
7. \(n - 35 = 40\); \(n = 75\) 8. \(n + 27 = 42\); \(n = 15\)
9. \(\frac{1}{2}n = 13\); \(n = 52\) 10. \(n^2 = 144\); \(n = 12\) or \(n = -12\)

Go Further
Student page 16 Encourage students to write their word equations in several forms.

Answers for student page 16: 11. Check students’ work.

Assessment
Student self-assessment page 16 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students solve simple equations by using inverse operations?
Materials
Student page 17
Overhead projector and transparency (optional)
Rulers
Chart paper and markers

Concepts and Handbook References
Study diagonals in rectangles. (MOC 345)
Make a table. (MOC 285)

Background
If the length of the rectangle is $l$ and its width is $w$, then the number of squares intersected ($i$) in the rectangle is: $i = l + w - \text{GCF}(l, w)$, where $\text{GCF}(l, w)$ is the greatest common factor of $l$ and $w$.

\[
\begin{array}{ccc}
  & 3 & 4 \\
1 & & \\
2 & & \\
\end{array}
\]

\[i = 3 + 2 - 1 = 4\]

Today's Challenge
Student page 17 Make sure students have very sharp pencils for this activity. Pass out rulers and have students draw the diagonals and record their data. Once they have completed their task, have them check with a friend to see if they have the same data.


Look Ahead
Make a chart like the one on student page 17 and fill in the answers for students to use as a reference next time.

Get Started
Tell students that you have to install an electrical wire in your tiled bathroom floor that goes from one corner of the room to the opposite corner. Since the wire has to go beneath the 12-inch $\times$ 12-inch tiles, you will need to rip some of them up, and you want to know how many need to be removed. Your bathroom is eight feet long and 12 feet wide. Draw an $8 \times 12$ rectangle on a square grid on the board (or overhead transparency) and draw a diagonal. Count the number of squares that were cut by the diagonal (16).
Materials
Student page 18
Large-grid graph paper (optional)

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Write an equation to describe a functional relationship. (MOC 205, 244)
Use the greatest common factor of two numbers. (MOC 066)
Use prime numbers. (MOC 058)

Background
A function is a relationship between two numbers, x and y, such that for each x, there is exactly one y. Two numbers are relatively prime if they have no common factors other than one. If the dimensions of a rectangle are not relatively prime, then the diagonal will pass through at least one lattice point in the diagram.

- dimensions: 4 × 6
- GCF of dimensions: 2
- dimensions with the GCF factored out: 2 × 3
- The diagonal passes through a lattice point where the 2 × 3 rectangles meet.

Get Started
Define relatively prime and ask students to give you examples of relatively prime numbers. (3, 5; 32, 7; 8, 21)

Answers for student page 18: Check students’ work. Possible answers are provided.

Things we discovered:
- The dimensions of A rectangles are relatively prime.
- When the rectangle’s dimensions are relatively prime, the number of intersections (i) is the sum of the length (l) and width (w) minus one. That is, i = l + w - 1.
- The dimensions of B rectangles have common factors greater than one.
- When the rectangle’s dimensions have a greatest common factor other than one, the number of intersections (i) is the sum of the length (l) and width (w) minus the greatest common factor of the length and width (GCF(l, w)). That is, i = l + w - GCF(l, w).

1. Add the length and width and subtract the greatest common factor of the length and width: i = l + w - GCF(l, w) 2. 17 3. 14 4. Rectangles whose dimensions have a common factor other than one will have at least one lattice point-diagonal intersection.

Assessment
Student self-assessment page 18 Have students circle one of the two choices to describe how they feel about this activity.
Assessment tip Do students know how to find the greatest common factor for two numbers and can they recognize it when it appears in a pattern such as this?
Materials
Student page 19

Concepts and Handbook References
Determine the lengths of the sides of quadrilaterals given sufficient information. (MOC 365)
Compute with metric and customary measures. (MOC 535, 536)

Get Started
Remind students that the perimeter of any polygon is the sum of the lengths of its sides. In a square the sides are equal, so the perimeter can be found by multiplying the length of one side by four. This means that the perimeter is a product with four as one of its factors. Ask students how to find the length of a side of a square if they know the perimeter. Point out that multiplication and division are inverse operations.

Student page 19 Have students use what they know about perimeter to answer the questions in the Get Started section.

Answers for student page 19:
1. 11 in.
2. 2.06 cm

Today's Challenge
Explain that today you will be playing a game called “Who Wants to Be the Top Scorer?” Have each student take a blank sheet of paper and draw a square near the top of the page. Have students choose a perimeter for this square from the numbers 10 through 12 and record their selection by writing Perimeter = ___ centimeters under their squares. Have students determine the side lengths of the square for the perimeter they have chosen, then ask them to number their papers from 1 through 5.

As you ask each question, have the students look at their squares and answer the question. Yes answers will score points.
1. Is the length of a side of your square a decimal number that contains at least one five? If yes, score 10 points.
2. Is the length of a side of your square 2.5 or three centimeters? If yes, score 5 points.
3. Is the length of a side of your square a decimal number that terminates in the tenths place? If yes, score 9 points.
4. Is the length of a side of your square a decimal number that terminates in a place to the right of the hundredths place? If yes, score 15 points.
5. Is the length of a side of your square greater than 2.625 and less than 2.875? If yes, score 8 points.

Have students find their total scores. Determine which student has the highest score. Have that student draw the square on the board and explain how he or she scored the points. If any student earned no points, have him or her explain the scoring, otherwise work together to create a zero-point square. (Possible zero-point square: perimeter 10.44 centimeters, side 2.61 centimeters)

Go Further
Student page 19 Have students solve the riddle and create riddles for a friend to solve. Have the solver sign his or her name.

Answers for student page 19:
3. 8.23 in., 16.46 in., 7.5 in., 7.5 in. 4–5. Check students’ riddles. Make sure that they include specific linear units, either customary or metric.

Assessment
Student self-assessment page 19 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students know how to determine the length of a side of a square, given its perimeter?
Materials
Student page 20
Overhead projector (optional)

Concepts and Handbook References
Use the Work Backward and Draw a Diagram problem-solving strategies. (MOC 479, 483)
Compute with rational numbers. (MOC 104, 132, 160, 187)

Get Started
Discuss how a picture can be worth 1000 words when solving problems. Point out that explanations do not always need to be in full sentences. Sometimes diagrams are far more effective in illustrating a process. Useful diagrams can even be doodled on scratch paper while taking a test. Pose this problem.

If I take a number and divide it by five, add $9 \div 20$ to the quotient, multiply the sum by four, then subtract $2 \div 5$ from the product, I get $2 \div 5$. What number did I start with?

Draw this backtracking diagram on the board or overhead projector, repeating the appropriate part of the problem as you write each operation.

Ask the class how they might use the diagram to solve the problem. Work through the problem backward, using inverse operations.

Student page 20 Have students work independently on the problem at the top of student page 20. Have them check their solution with a friend, then discuss their solutions in class. (You started with 15 boxes of candy.)

Today's Challenge
Student page 20 Encourage students to read carefully, setting up the backtracking diagrams one sentence at a time.

Answers for student page 20: 1. 70 mph 2. $y = 8$
3. Check students' advice. They might suggest using the strategy for problems in which the final answer and steps used to get there are known but the starting point is not.

Go Further
On a separate sheet of paper, have each student write his or her name and create one problem similar to those on student page 20. They should write the correct answer on the back. Have them share their problem with a friend, who will draw a diagram to solve the problem, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 20 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students consistently use inverse operations as they work backward?
Materials
Student page 21
Math Maze Cards (Week 5 Activity 21)

Concept and Handbook Reference
Review geometry vocabulary. (MOC 572)

Background
While there are many two-dimensional figures that are not polygons (for example, circles), much of the study of geometry at this level relates to polygons. Polygons are named using prefixes and suffixes that are clues to their shapes.

tri- three oct- eight
quad- four poly- many
pent- five -lateral side
hex- six -angular angle
hept- seven

It is very important for students not to assume that a figure has attributes unless the figure is named or labeled to show attributes.

Examples:
• What is special about a square?
   Because the figure has been named a square, you can assume it has four right angles, four congruent sides, and two pairs of parallel sides.

• Name this figure.
   The figure is marked to show two congruent angles and two pairs of adjacent congruent sides. Since no sides are marked parallel , you cannot assume any sides are parallel. It is a kite

Get Started
Help students define geometry as the study of shapes and space. Specifically define polygon as a two-dimensional (flat) figure made up of line segments that meet only at their endpoints. Draw this table on the chalkboard.

<table>
<thead>
<tr>
<th>Polygons</th>
<th>Not Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ □</td>
<td></td>
</tr>
<tr>
<td>□ △ □</td>
<td></td>
</tr>
<tr>
<td>△ □ △</td>
<td></td>
</tr>
<tr>
<td>△ □ □ □</td>
<td></td>
</tr>
</tbody>
</table>

Encourage students to add to your table, and then discuss how to be specific when naming polygons. Ask, What is the name of a polygon with four sides? (quadrilateral is the general name but students may also give specific types such as parallelogram, square, rectangle, rhombus, trapezoid, kite)

Today’s Challenge
Distribute the 18 Math Maze cards for week 5. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 185.

Student page 21 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity.

Answers for student page 21: 1. f 2. i 3. j 4. l 5. b 6. m 7. d 8. c 9. h 10. k 11. a 12. e 13. g

Go Further
Student page 21 Encourage students to be as specific as possible when naming these polygons.

Answers for student page 21: 14. rhombus 15. hexagon 16. right triangle 17. trapezoid

Assessment
Student self-assessment page 21 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students easily distinguish among quadrilaterals by looking at special side- and angle-relationships?
Materials
Student page 22
Number cubes (1–6)
Chart paper and markers (optional)

Concepts and Handbook References
Make a sample space. (MOC 463–464)
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Determine whether a game is fair. (MOC 465–466)

Get Started
Tell students that they will be working in groups of three to collect data about a game. Be sure that they understand the rules of the game (student page 22), then discuss how to record class data without being disruptive. Draw this table on the board or chart paper and use it to collect class data. Students will need this to answer exercise 1.

<table>
<thead>
<tr>
<th>Tallies</th>
<th>Player A Wins</th>
<th>Player B Wins</th>
<th>Player C Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Today’s Challenge
Student page 22 Distribute two 1–6 number cubes to each group of students. Have them play two games and record their results on the chart up front after the games but before they try to answer exercise 1.

Answers for student page 22: 1. Accept any answer as long as the answer to exercise 3 reflects this fact: Player B is theoretically most likely to win, so the game is not fair.

<table>
<thead>
<tr>
<th>×</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
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<td>3</td>
<td>4</td>
<td>5</td>
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<td>6</td>
<td>8</td>
<td>10</td>
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<tr>
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<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
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<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
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<tr>
<td>6</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product</th>
<th>Number of Ways to Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
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<tr>
<td>4</td>
<td>3</td>
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<tr>
<td>5</td>
<td>2</td>
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<tr>
<td>6</td>
<td>4</td>
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<td>8</td>
<td>2</td>
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<tr>
<td>9</td>
<td>1</td>
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<td>10</td>
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<td>16</td>
<td>1</td>
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<td>20</td>
<td>2</td>
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<td>24</td>
<td>2</td>
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<tr>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>36</td>
</tr>
</tbody>
</table>

3. This is not a fair game, Player A’s probability of winning is $\frac{13}{36}$, Player B’s probability of winning is $\frac{17}{36}$ and Player C’s probability of winning is $\frac{6}{36}$.

Look Ahead
If students are not convinced of the unfairness of the game, have them try it 10 more times before next time.
**Materials**
Student page 23

**Concept and Handbook Reference**
Use simple probability to create a fair game.
(MOC 461–469)

**Get Started**
Put students back into their groups of three. Be sure that they have their results from student page 22.

**Go Further**
**Student page 23** Encourage students to use the fact that the sum of the probabilities of the outcomes for an event is always one as they try to create a fair game.

**Answers for student page 23:** Check students’ work. Possible answers are provided.

**Things we discovered:**
- The products 6 and 12 occur four times, which is the most for any product.
- The products 1, 9, 16, 25, and 36 occur only once, so they are least likely to occur.
- The products 2, 3, 5, 8, 10, 15, 18, 20, 24, and 30 occur twice.
- The product 4 occurs three times.
- The products 7, 11, 13, 14, 17, 19, 21, 22, 23, 26, 27, 28, 29, 31, 32, 33, 34, and 35 cannot be rolled.

1. \( \frac{12}{36} \) or \( \frac{1}{3} \)
2. There are many fair games that could be created. Here are two such games. Game 1: Player A 1, 3, 4, 6, 9, and 25; Player B 2, 5, 8, 10, 15, and 18; and Player C 12, 16, 20, 24, 30, and 36. Game 2: Player A 6, 9, 10, 15, 18, and 25; Player B 2, 5, 8, 20, 24, and 30; and Player C 1, 3, 4, 12, 16, and 36.

3. There are many ways to create a fair game. For example, if you have a fair game, just swap a number that has two outcomes with some other number that has two outcomes.

**Assessment**
**Student self-assessment page 23** Have students circle one of the two choices to describe how they feel about this activity.

**Assessment tip** Can students create a sample space to help them determine the probability of events?
Materials
Student page 24
Blank paper (heavyweight if possible) or index cards
Box

Concept and Handbook References
Determine missing dimension of a rectangular prism
when the volume and two other dimensions are
given. (MOC 397, 402, 565)

Get Started
Remind students that the volume of a rectangular
prism is the number of cubic units inside the prism.
Model this with any available box, such as a tissue
box. The volume is found by multiplying the length
by the width by the height.

\[ h = \text{height} \]
\[ l = \text{length} \]
\[ w = \text{width} \]

Draw a rectangular prism on the board. Label the
length 5 units, the width 3 units, and the height 2
units. Have the students find the volume. (30 cubic
units) Then have them explain how they solved the
problem. (Multiply length by width by height;
\[ 5 \times 3 \times 2 = 30. \])

Student page 24 Have students complete the two
examples at the top of student page 24. Ask them
to explain how the missing dimensions can be
found. (Volume = \( lwh \); substitute known values,
then simplify.)

For example 1,
\[ 4 \times 2 \times h = 40 \]
\[ 8h = 40, h = 5 \text{ centimeters} \]

For example 2,
\[ 2 \times w \times 2 = 40 \]
\[ 4w = 40; w = 10 \text{ centimeters} \]

Today's Challenge
Student page 24 Have students complete the table
on student page 24. Explain that each row has two
prisms that have the same volume but different
dimensions. In each case, students are to find the
missing dimension.

Answers for student page 24: 1. 4 cm 2. 12 cm
3. 4 yd 4. 1 yd 5. 2 in. 6. 1 in. 7. 4 km 8. 16 km
9. 5 ft 10. 2 ft 11. 3 mm 12. 4 mm

Go Further
Have pairs of students make a set of cards to play
the game “Concentration.” Each pair of students
will need 12 small pieces of paper or 12 index
cards. Have the students use one slip of paper or
card to copy the diagrams from the right side of
each box in the table on student page 24, including
the units of measure. Students will not copy the
information about the volume or the two examples
at the top of the page.

Instructions for playing “Concentration” Shuffle the
cards and lay them facedown in a \( 3 \times 4 \) array. The
first player turns over any two cards. If the cards
match (have the same volume), the player keeps
the cards and goes again. If the cards do not
match, the player turns the cards back over and the
other player takes a turn. Play continues until all
cards have been taken. The player with more cards
at the end of the game wins.

Assessment
Student self-assessment page 24 Have students cir-
cle one of the two choices to describe how they feel
about this activity.

Assessment tip Can students find a missing dimen-
sion of a rectangular prism when given two dimen-
sions and the volume?
Materials
Student page 25

Concepts and Handbook References
Add and subtract fractions and mixed numbers. (MOC 104, 132)
Use benchmark fractions to estimate sums and differences. (MOC 032)

Get Started
Estimating can help in two ways when students are taking a test. First, it can alert them when something about their computation does not make sense. Second, it can help them to choose a correct answer from among several choices without ever completing the computation. Ask students to figure out a ballpark difference for $4\frac{1}{2} - 1\frac{1}{2}$. Since $\frac{1}{2} > \frac{1}{2}$ and $4 - 1 = 3$, the difference is a little less than three.

Write these expressions on the board and discuss making reasonable estimates for the value of each.
- $3\frac{1}{2} + 2\frac{3}{7}$ (a number close to but still less than six, since $2 + 3 = 5$ and $\frac{1}{2}$ and $\frac{3}{7}$ are both less than $\frac{1}{2}$)
- $\frac{5}{9} + \frac{3}{3}$ (a number a little greater than one, since $\frac{5}{9}$ and $\frac{3}{3}$ are both a little greater than $\frac{1}{2}$)
- $4\frac{1}{2} - 2\frac{1}{8}$ (a number a little greater than two, since $4 - 2 = 2$ and $\frac{1}{2} > \frac{1}{8}$)

Student page 25 Work with the class on the problem at the top of student page 25. Have them explain why each choice was or was not acceptable. (Choice C is correct. Choices A and B are too great, since $\frac{5}{8} < \frac{5}{8}$ and $5 - 2 = 3$. Choice D is not great enough, since $\frac{5}{2}$ is a little less than one half, and $\frac{5}{8}$ is a little more than one half, so the difference should be closer to three than to two.)

Today's Challenge
Student page 25 Have students work independently on student page 25. Encourage them to use benchmark fractions for their estimates.


Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 25. Encourage them to be sure that the correct answer can be distinguished by estimating. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 25 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students estimate confidently?
Materials
Student page 26
Math Maze Cards (Week 6 Activity 26)

Concept and Handbook References
Review equivalent fractions and decimals.
(MOC 043-045, 538)

Get Started
Discuss equivalent forms of the same number. Ask
students to generate multiple forms of the fraction \(\frac{1}{2}\)
(\(\frac{2}{4}, \frac{4}{8}, \text{etc.}\)), and then ask for decimal equivalents
(0.5, 0.50, 0.500, etc.). Compare a decimal equivalent to one of the equivalent fractions (0.5 = \(\frac{1}{2}\)). Next, discuss comparing fractions and decimals by finding equivalents or by using benchmarks of 0, \(\frac{1}{2}\), and 1.

Example: Compare \(\frac{1}{4}\) to 0.7
• You could reason that since \(\frac{1}{4}\) is less than half and
0.7 is greater than half, \(\frac{1}{4} < 0.7\).
• You could write an equivalent decimal for \(\frac{1}{4}\) and
compare: 0.25 < 0.7.
• You could write an equivalent fraction for 0.7 and
write each fraction with a common denominator:
\(\frac{1}{4} = \frac{25}{100}\) and \(0.7 = \frac{70}{100}\).

Review common repeating decimals and how to
write them. Students should be familiar with decimal equivalents for thirds and ninths. Have students discuss the patterns found in the following list.

\[
\begin{align*}
\frac{1}{3} & = 0.\overline{3} \\
\frac{2}{3} & = 0.\overline{6} \\
\frac{1}{9} & = 0.\overline{1} \\
\frac{2}{9} & = 0.\overline{2} \\
\frac{3}{9} & = 0.\overline{3} \\
\frac{4}{9} & = 0.\overline{4} \\
\frac{5}{9} & = 0.\overline{5} \\
\frac{6}{9} & = 0.\overline{6} \\
\frac{7}{9} & = 0.\overline{7} \\
\frac{8}{9} & = 0.\overline{8} \\
\end{align*}
\]

Today's Challenge
Distribute the 18 Math Maze cards for week 6. Each
student should receive at least one card, but since
all cards need to be distributed, some students may
need to get more than one card. Use the cards to
play the Math Maze game.

Instructions for playing Math Maze Ask students to
look at their cards. Ask one student to read the
question that is written on his or her card. Next ask,
"Who has the card with the answer to the question
just read?" Ask that student to read the answer, and
then read the question on his or her card. Play con-
tinues until all questions have been answered. The
last answer to be read should be the answer on the
first student’s card.

The correct sequence of questions and answers is
shown on page 186.

Student page 26 When the group has finished play-
ning the game, have students open their books and
complete the Today's Challenge activity.

Answers for student page 26: 1. < 2. = 3. > 4. > 5. < 6. > 7. = 8. < 9. < 10. =

Go Further
Student page 26 Have students complete this sec-
tion of the student page.

Answers for student page 26: 11. 0.8 12. \(\frac{3}{4}\)
13. 0.375 14. \(\frac{1}{4}\) 15. \(\frac{2}{5}\)

Assessment
Student self-assessment page 26 Have students cir-
cle one of the two choices to describe how they feel
about this activity.

Assessment tip Are students able to recall the
equivalents for common decimals and fractions?
Materials
Student page 27
Calculators
Chart paper or overhead transparency and markers

Concepts and Handbook References
Write fractions as decimals. (MOC 044)
Look for patterns in repeating decimals whose fraction forms have the same denominators.
(MOC 023, 538)

Background
If a fraction is written as a decimal, it will either terminate or repeat. If it repeats, the repeating pattern will be complete before the number of decimal places is equal to the denominator of the fraction. This occurs because when you divide by the number \( d \), there are only \( d \) possible remainders, zero through \( d - 1 \). When a remainder repeats, the decimal portion of the quotient will repeat. The fraction \( \frac{1}{7} \) has six parts in its repeating decimal equivalent. Another interesting fact is that the number of places in the repeating decimal is always a factor of the number that is one less than the denominator. For example, the sevenths have a six-place repeat, as do the thirteenths. The elevenths have a two-place repeat.

Get Started
Have four volunteers come to the board and use long division to find the decimal equivalents of these fractions: \( \frac{1}{3}, \frac{2}{9}, \frac{3}{11}, \) and \( \frac{5}{6} \). Ask them what is unusual about the division. (They should say that the division process keeps repeating itself.) Discuss the fact that \( \frac{1}{3} = 0.33 \), since the division does not end after two places. Tell them that you indicate the part of a decimal that repeats by drawing a bar, called the vinculum, over the top of it.

Today’s Challenge
Student page 27. Distribute calculators to groups of three students. Remind them that their calculators will show the same number of places for a repeating decimal, regardless of the length of the repeat. They will need to study the display to discover the repeat.

Answers for student page 27: 1. 0.142857
2. 0.285714 3. 0.428571 4. 0.571428
5. 0.714285 6. 0.857142 7. 1.142857
8. 1.285714 9. 1.428571 10. 6 11. 1.571428
12. 0.09 13. 0.18 14. 0.27 15. 0.16 16. 0.3
17. 0.6 18. 0.83 19. 1.16 20. 1.3 21. 0.076923
22. 0.153846 23. 0.230769 24. 0.307692
25. 0.384615 26. 0.461538 27. 0.538461
28. 0.615384 29. 0.692307 30. 0.769230
31. 0.846153 32. 0.923076

Look Ahead
Prepare a chart showing the answers for exercises 1–9 and 12–32 on student page 27. Students will need to have accurate data for next time.
Materials
Student page 28
Calculators

Concepts and Handbook References
Work with repeating decimals. (MOC 023)
Use the Look for a Pattern problem-solving strategy (MOC 484)

Get Started
Post the answers to student page 27 to ensure that everyone has accurate data to work with today.

Go Further
Student page 28 Have students work in pairs to study student page 27 and write their analyses. If students have trouble with exercise 5, give them calculators and have them repeat exercise 1 for \(\frac{2}{13}\) and \(\frac{4}{9}\).

Answers for student page 28: Check students' work. Possible answers are provided.

Things we discovered:
- There are two six-digit representations for thirteenth. One uses the digits 0, 7, 6, 9, 2, and 3 and the other uses the digits 1, 5, 3, 8, 4, and 6.

\[
\begin{align*}
\frac{1}{13} + \frac{1}{13} & = \frac{2}{13} \\
0.076923 + 0.076923 & = 0.153846
\end{align*}
\]

set one digits \hspace{2cm} set two digits

\[
\begin{align*}
\frac{2}{13} + \frac{1}{13} & = \frac{3}{13} \\
0.153846 + 0.076923 & = 0.230769
\end{align*}
\]

set one digits

\[
\begin{align*}
\frac{3}{13} + \frac{1}{13} & = \frac{4}{13} \\
0.230769 + 0.076923 & + 0.307692
\end{align*}
\]

set one digits

\[
\begin{align*}
\frac{4}{13} + \frac{1}{13} & = \frac{5}{13} \\
0.307692 + 0.076923 & = 0.384615
\end{align*}
\]

set two digits

- The same six digits (1, 4, 2, 8, 5, 7) appear over and over in the decimal representations of sevenths.
- All decimal representations of rational numbers seem to terminate or repeat before the number of places is equal to the denominator. For example, \(\frac{1}{3}\) repeats after one decimal place, \(\frac{1}{7}\) repeats after six decimal places, and \(\frac{3}{13}\) repeats after six decimal places.
- The number of decimal places in the repeating part of the decimal is always a factor of the number that is one less than the denominator. For example, \(\frac{1}{11}\) repeats after two places, which is a factor of 11 – 1 = 10. The fraction \(\frac{1}{13}\) repeats after six places, which is a factor of 13 – 1 = 12.

1. A. 6 B. 6 C. 1, 2, 3, 6 2. A. 2 b. 10 C. 1, 2, 5, 10 3. A. 1 B. 5 C. 1, 5 4. A. 6 B. 12 C. 1, 2, 3, 4, 6, 12 5. The number of places in the repeat is a factor of \(n – 1\).

Assessment
Student self-assessment page 28 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students perceive patterns in repeating decimals?
Materials
Student page 29
Math Jumble activity poster and digit cards

Concepts and Handbook References
Solve equations. (MOC 243)
Graph solutions to equations. (MOC 247)

Background
Ordered pairs are normally written (x, y). The ordered pair (2, 5) satisfies (is a solution to) the equation $2x + 1 = y$ because when two is substituted for x and five is substituted for y, the equation is true: $2(2) + 1 = 5$.

Get Started
Write these equations on the board.
- $x + 2 = y$
- $2x + 5 = y$
- $4x - 3 = y$

Have students give three ordered pairs that would satisfy each equation. Students will likely choose “nice” values for x such as one, two, and three. Encourage them to try negative integers as well.

[Possible solutions:
$x + 2 = y \rightarrow (1, 3), (0, 2), (-3, -1)$;
$2x + 5 = y \rightarrow (0, 5), (2, 9), (-2, 1)$;
$4x - 3 = y \rightarrow (-3, -15), (-1, -7), (2, 5)$]

Today’s Challenge
Use the 0–9 digit cards to construct this poster.

Explain that the object of today’s Math Jumble is to find ordered pairs that are solutions to the equation $2x + 3 = y$. Students must supply the negative sign if negative integers are used.

Ordered pairs can be made with any two or more adjoining digits (horizontally, vertically, and/or diagonally) on the poster. Digits on the poster can be used more than once, but not in the same ordered pair. For example, the first two digits in the top row can be used to make the ordered pair (1, 5) which is a solution of $2x + 3 = y$. Record the ordered pairs students make.

Possible ordered pairs: (5, 13), (3, 9), (-4, -5)

Today’s Challenge
Student page 29 Have students use the Math Jumble on student page 29 to find strings of digits that can be used to make ordered pairs that are solutions to the given equation. Students must supply the negative sign for negative integers.

Answers for student page 29: Check students’ work. Possible answers are provided. 1. (1, 5), (2, 8), (3, 11), (-1, -1), (-2, -4), (-3, -7)

Go Further
Student page 29 Have students use the coordinate grid on the student page to plot the ordered pairs found in exercise 1.

Answers for student page 29: 2. Check students’ graphs. 3. The ordered pairs should be collinear and represent the graph of $y = 3x + 2$.

Assessment
Student self-assessment page 29 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students substitute values for x into an expression? Can they determine whether an ordered pair is a solution of an equation?
Materials
Student page 30
Rulers

Concept and Handbook References
Use Venn diagrams to organize data. (MOC 364)

Background
Many standardized tests have incorporated the use of constructed-response items. These problems ask students to explain their solutions. Scores on these items depend on how well the solutions are explained.

Get Started
Venn diagrams can be a useful tool for organizing information to help students solve problems. Post this diagram and table side by side on the board. Discuss how the information in the table fits in the diagram.

![Venn Diagram]

Sports Preference Survey

<table>
<thead>
<tr>
<th></th>
<th>Soccer</th>
<th>Basketball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer</td>
<td>🏈</td>
<td>🏀</td>
</tr>
<tr>
<td>Basketball</td>
<td>🏀</td>
<td>🏈</td>
</tr>
<tr>
<td>Only Soccer or Only Basketball</td>
<td>🏈 🏀</td>
<td>🏃️</td>
</tr>
<tr>
<td>Neither</td>
<td>🏃️</td>
<td>🏃️</td>
</tr>
<tr>
<td>Total votes</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Ask and discuss these questions.
- How many students liked both soccer and basketball? (2)
- How many students liked only soccer? (3)
- How many students liked only basketball? (7)

Student page 30 Start the problem at the top of page 30 with the students. Go through filling in the diagram and finding the answer to A. Have them work on B–D with a partner. Discuss answers before starting Today’s Challenge. (A. 24 B. 19 C. 1 D. 2)

Math

19
7
12
13

Science

Today’s Challenge
Student page 30 Encourage students to pay careful attention to how categories are described. Both clearly indicates that a number belongs in two categories. However, saying a student can identify France on the map is not saying that student can only identify France on the map.

Answers for student page 30: 1. 3 2. 2 3. 6 4. 7 5. 4 6. 12 7. Check students’ advice. They should mention paying careful attention to the words and, both, neither, and only.

Go Further
On a separate sheet of paper, have each student write his or her name and create a problem similar to those on student page 30. It might be a good idea for them to work backward from a filled-in Venn diagram to create a problem that works. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problem and sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 30 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Can students use a Venn diagram to organize information?
Materials
Student page 31
Math Maze Cards (Week 7 Activity 31)

Concept and Handbook Reference
Add integers. (MOC 108)

Background
The set of integers is the set of whole numbers, their opposites, and zero.
Integers = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}
Integers can be modeled on a number line.

You can use a number line to add integers. Start at the first addend.

- When you add a positive number move to the right.
- When you add a negative number move to the left.

Get Started
Talk about situations in which students have added integers such as receiving $10 and spending $7 (10 + -7 = 3); the temperature dropping 4° and then rising 6° (-4 + 6 = 2); gaining 13 yards on a play followed by losing 8 yards on the next play (13 + -8 = 5).

Ask students to do the following problems. In each case model the problem on a number line.

- What is \(-4 + -4\)? \((-8\)

- What is \(6 + -5\)? \((1\)

- What is \(-7 + 5\)? \((-2\)

Today's Challenge
Distribute the 18 Math Maze cards for week 7. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, "Who has the card with the answer to the question just read?" Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 187.

Student page 31 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 31 in the student book.

Answers for student page 31: 1. -26; -21; -11; -9
2. -20; -15; -5; 3. -3; -9; -4; 6; 8

Go Further
Student page 31 Students will devise addition expressions that can be illustrated on the blank number lines.

Answers for student page 31: 4-5. Check students’ work.

Assessment
Student self-assessment page 31 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students consistently use mental math to find the sum of two integers or do they depend on the number line?
Materials
Student page 32
Checkerboard

Concepts and Handbook References
Use the Look for a Pattern and Make a List problem-solve strategies. (MOC 484, 480)
Use powers of two. (MOC 071)

Get Started
Hold up a checkerboard for a few seconds and then hold it behind your back. Ask students how many squares are on the board. After students guess, hold it up so students can check their guesses. Ask students how many pennies the board will hold with one penny per square (64).

Today's Challenge
Student page 32 Here are some hints for students who are having difficulty.
• Ask students what mathematical procedure is related to doubling the total number of pennies on each square (finding powers of two).
• After finding the number of pennies on the first ten squares, suggest that students might look for a pattern to find the pennies on subsequent squares.

Answers for student page 32: 1. 64; 128 2. Accept any answer

3.

<table>
<thead>
<tr>
<th>Square Number</th>
<th>Number of Pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
</tr>
<tr>
<td>8</td>
<td>128</td>
</tr>
<tr>
<td>9</td>
<td>256</td>
</tr>
<tr>
<td>10</td>
<td>512</td>
</tr>
<tr>
<td>11</td>
<td>1,024</td>
</tr>
<tr>
<td>12</td>
<td>2,048</td>
</tr>
<tr>
<td>13</td>
<td>4,096</td>
</tr>
<tr>
<td>14</td>
<td>8,192</td>
</tr>
<tr>
<td>15</td>
<td>16,384</td>
</tr>
<tr>
<td>16</td>
<td>32,768</td>
</tr>
<tr>
<td>17</td>
<td>65,536</td>
</tr>
<tr>
<td>18</td>
<td>131,072</td>
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<td>19</td>
<td>262,144</td>
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<td>524,288</td>
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</tr>
<tr>
<td>22</td>
<td>2,097,152</td>
</tr>
<tr>
<td>23</td>
<td>4,194,304</td>
</tr>
<tr>
<td>24</td>
<td>8,388,608</td>
</tr>
<tr>
<td>25</td>
<td>16,777,216</td>
</tr>
</tbody>
</table>

4. not likely, explanations will vary.

Look Ahead
Tell students that next time they will compare their results and discuss how to use the calculator to find the number of pennies on any square by using a feature of the calculator.
Go Further
Student page 33 Encourage students to explore the capabilities of their calculators. The constant function and the $y^x$ key will speed up the multiplication. The $y^x$ key will allow them to find the number of pennies on any square, in any order. This means that they can find the number of pennies on square 10 by computing $2^{10} - 1$.

Answers for student page 33: Check students’ work. Possible answers are provided.

Things we discovered:
- The number of pennies on each square can be represented as a power of two. Square one is $2^0$, square two is $2^1$, square three is $2^2$, and so forth.
- You can use the power key on the calculator to find the number in any square without using two as a factor the number of times indicated by the square number.

  1. Each square ($n$) holds $2^{n-1}$ pennies. 2. The last square holds $2^{30}$ pennies. 3. That is more than $92,000,000,000,000,000$. 4. There are not enough places in the calculator display for $2^{30}$, so it is truncated and written in scientific notation.

Assessment
Student self-assessment page 33 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students use the $y^x$ and constant functions on their calculators?
Materials
Student page 34
Slips of paper containing integers -10 through 10

Concept and Handbook References
Use mental math with negative numbers.
(MOC 108, 136, 164, 193)

Background
You can use a number line to add or subtract numbers.
• To add a positive number, move right from the first addend.
• To add a negative number, move left from the first addend.
• To subtract a positive number, move left from the first addend.
• To subtract a negative number, move right from the first addend.

To multiply or divide integers, compute as for whole numbers, then think of the three elements of the computation as two factors and a product.
• factor₁ × factor₂ = product
• product ÷ factor₁ = factor₂ or product ÷ factor₂ = factor₁
• If both factors are positive, so is the product.
• If both factors are negative, the product is positive.
• If one factor is positive and one is negative, the product is negative.

Get Started
Review the rules for operating on integers. Students will be expected to use mental math as they compute with negative numbers in this activity.

Today's Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with an integer from -10 through 10. You do not need to use all of the numbers, but be sure that one student receives the number -4, since that number will be the “Fantastic Finalist.” For a larger group, additional numbers less than -10 and greater than 10 may be used.

Have all students hold their numbers and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
• If your number is less than three units from five, sit down. (3, 4, 5, 6, 7)
• If your number is greater than 4 - 3, sit down. (8, 9, 10)
• If your number is less than 3 + 7, sit down. (-5, -6, -7, -8, -9, -10)
• If your number is greater than 2 x 2, sit down. (-3, -2, -1, 0, 1, 2)

At this point, only the student holding the number -4 should still be standing. That student is the Fantastic Finalist.”

Go Further
Student page 34 Have students complete the activity on the student page and create a “Fantastic Finalist” activity for a friend to solve.

Answers for student page 34:
1-5.

6. 2. 7. Check students’ work.

Assessment
Student self-assessment page 34 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students use mental math to compute with negative numbers?
Materials
Student page 35

Concepts and Handbook References
Estimate and use logical reasoning to select the correct answer from among four choices. (MOC 528)
Compute with decimals. (MOC 184–186)

Get Started
Tell students that what they know about how division with mixed numbers works can help them choose the correct answer from among a set of possibilities even without doing all of the computation. Introduce today's activity by displaying the following expression and answer choices. Ask them to reason logically to analyze each answer choice.

1730.88 ÷ 3.6
A. 4808 (No; greater than dividend)
B. 480.8 (Yes; 1600 ÷ 4 = 400)
C. 48.08 (No; too small)
D. 4.808 (No; too small)

Student page 35 Work with the class on the exercise at the top of student page 35. Teri is correct and the reasoning should involve relative size of the quotient.

Today's Challenge
Student page 35 Have students work independently on student page 35. Ask them to choose the correct answer and then explain their reasoning for each choice.

Answers for student page 35: Check students' explanations. Samples are given. 1. C; A and D are too small, and B is too great 2. A; B is too great; both C and D are too small 3. B; A and D are too small and C is too great. 4. A; B is too small; C and D are too great 5. Check students' advice. They might mention the fact that just knowing the relative size of the quotient (tens, hundreds, thousands) may be enough to make a correct choice.

Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 35. They should write the correct answers on the back. Have them share their problems with a friend, who will reason out (instead of computing) the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 35 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students trust their ability to estimate well enough to choose the correct quotient for division with decimals from among several distracters?
Math Maze

Materials
Student page 36
Math Maze Cards (Week 8 Activity 36)

Concept and Handbook References
Find least common multiple and greatest common factor of numbers and expressions. (MOC 066, 068)

Background
• One way to find least common multiple:
  Make a list of multiples for each number. Stop when you find a multiple in one list that matches a number in the other list.
• One way to find greatest common factor:
  Make a list of all of the factors for each number. Select the greatest number that appears in each list. If the expressions involve variables, select any common variable factors as well.

Get Started
Discuss the difference between multiples and factors. Discuss a strategy for finding the least common multiple of two numbers. Discuss a strategy for finding the greatest common factor of two numbers or two expressions. Try these examples.
• Find the greatest common factor of 18 and 10. (2)
• Find the greatest common factor of 20a and 35a. (5a)
• Find the least common multiple of 9 and 12. (36)
• Find the least common multiple of 15 and 10. (30)

Today's Challenge
Distribute the 18 Math Maze cards for week 8. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 187.

Student page 36
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 36 in the student book.

Answers for student page 36: 1. 7a  2. 30  3. 12  4. 5  5. 4a  6. 24  7. 3a  8. 45  9. 36  10. 9  11. 24  12. 12

Go Further
Student page 36
Have students study Today’s Challenge answers, then answer the Go Further questions.

Answers for student page 36: 13. n  14. m.

Assessment
Student self-assessment page 36
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Can students consistently and efficiently find the least common multiple and greatest common factor of two numbers or two expressions?
Materials
Student page 37
Geometric solids (Cube, rectangular prism, square pyramid, triangular pyramid, etc. if available)

Concepts and Handbook References
Discover characteristics of solid figures. (MOC 393–408)
Practice spatial visualization. (MOC 400, 405)

Background
A polyhedron is a three-dimensional figure with polygons and their interiors as faces. No polyhedron has any curved surfaces or edges.

Get Started
Display a cube. Ask students to identify an edge, a face, and a vertex. Note that the plural of vertex is vertices.

![Diagram of a cube with labels for vertex, face, and edge]

Describe the cube as an example of a polyhedron. Ask students to find the number of edges, faces, and vertices in the cube. (12, 6, 8)

Today’s Challenge
Student page 37 Discuss ways to look for classroom examples of polyhedrons (also called polyhedra) without disrupting the class. Here are some hints for successful data gathering.

- Be sure to count every face, vertex, and edge once and only once. You might mark each part of the polyhedron as it is counted.
- Polyhedrons do not need to have a simple name, or regular features. This composite figure is also a polyhedron.

Answers for student page 37: 1.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Faces</th>
<th>Number of Vertices</th>
<th>Number of Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Octahedron</td>
<td>8</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Truncated Pyramid</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Square Pyramid</td>
<td>5</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Triangular Prism</td>
<td>5</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

2. Check students’ work.

Look Ahead
Tell students that next time they will use today’s data to discover Euler’s Formula for polyhedrons.
Materials
Student page 38

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Discover characteristics of solid figures. (MOC 393–408)
Practice spatial visualization. (MOC 400, 405)
Write an equation to describe a functional relationship. (MOC 205, 244)

Get Started
Have students post their data for each polyhedron on the board. You can verify that the student data are accurate by checking that the figures fit Euler’s Formula \((F + V = E + 2)\). Do not reveal the formula to students at this time.

Go Further
Student page 38 Encourage students to write their equations in more than one form.

Answers for student page 38: Check students’ work. Possible answers are provided.

Things we discovered:
• For pyramids, the number of faces is the same as the number of vertices.
• There are always more edges than faces or vertices.
• The number of faces does not predict the number of vertices.
• The number of edges is two less than the sum of the faces and vertices.

1. Possible answers: \(F + V - 2 = E\) or \(F + V = E + 2\) or \(F = E + 2 - V\), or any version of this equation. Do not require a correct answer here if the equation is corrected in exercise 3.
2. A. Figure 1: \(F = 8, V = 12, E = 18\); Figure 2: \(F = 8, V = 12, E = 18\)
B-C. Check against exercise 1.
3. Check students’ work.

Assessment
Student self-assessment page 38 Have students circle one of the two choices to describe how they feel about this activity.
Assessment tips Did students derive a form of Euler’s Formula from their data?
Materials
Student page 39

Concepts and Handbook References
Identify perfect squares less than 200.
(MOC 076, 083)
Find square roots of perfect squares less than 200.
(MOC 076-077)

Background
The expression $x^2$ is read $x$ to the second power or $x$ squared. When a number is multiplied by itself, the product is the square of the number. The expression $\sqrt{x}$ is read the square root of $x$. By definition, $\sqrt{x}$ equals the positive number which, when multiplied by itself, is equal to $x$, even though opposites have the same square. ($4^2 = 16$ and $(-4)^2 = 16$; $\sqrt{16} = 4$).

Get Started
Write the following list of numbers on the board: 1, 4, 9, 16, 25, 36, 49, ... Ask students to describe the pattern. These numbers are called perfect squares. When a whole number is squared, you get a number in this list. If you find the square root of each of the numbers in the list, the result is an integer. Ask and discuss these questions.
- What is the value of 7? (49)
- What is the value of 13? (169)
- What is the value of $\sqrt{81}$? (9)
- What is the value of $\sqrt{144}$? (12)
- When an integer is raised to the third power, the result is a perfect cube. Ask students to list the first five positive perfect cubes. (1, 8, 27, 64, 125)

Student page 39
Have students use what they know about perfect squares to answer the questions in the Get Started section of student page 39.

Answers for student page 39:
1.

<table>
<thead>
<tr>
<th>$n$</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^2$</td>
<td>16</td>
<td>36</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>144</td>
</tr>
</tbody>
</table>

2.

<table>
<thead>
<tr>
<th>$\sqrt{n}$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
</table>

Today's Challenge
Explain that today the class will be playing a game called “Who Wants to Be the Top Scorer?” Have each student take a blank sheet of paper and write any perfect square less than 200, then ask them to number their papers from 1 through 5.

As you ask each question, have students look at their numbers and answer the question. Yes answers will score points.
1. Is your number odd? If yes, score 10 points.
2. Is the sum of your digits even? If yes, score 5 points.
3. Is your number greater than 5? If yes, score 9 points.
4. Is the digit in the ones place equal to the digit in the tens place? If yes, score 8 points.
5. Is your number a perfect cube? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student write his or her number on the board and explain how the points were scored. If any student earned no points, ask for an explanation of the scoring, otherwise, work together to create a zero-point number. (Possible zero-point numbers: 16, 36)

Go Further
Student page 39
Have students solve the riddles and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 39: 3. 64  4. 121  5. Check students’ riddles.

Assessment
Student self-assessment page 39
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Can students square and cube a whole number, and find the square root of a perfect square less than 200?
Materials
Student page 40
Chart or overhead transparency of blank fill-in grid (save for future activities)

Concepts and Handbook References
Compute with decimals. (MOC 158–159)
Use a machine-scoring style fill-in grid. (MOC 529)
Estimate to eliminate incorrect answer choices. (MOC 528)

Get Started
Display the blank fill-in grid and demonstrate its use. Be sure students understand that they write their answer in the top row, then fill in the digit, decimal point, or fraction bar that matches what they have written at the top of each column.
Talk about how it may be more efficient to estimate than to spend a lot of time computing during a timed test. If you can choose the correct answer by estimating, it is obvious that you understand the concept being tested.

Student page 40 Work with students on the problem at the top of student page 40. Ask for a volunteer to eliminate one of the answers and explain his or her reasoning. If members of the class do not agree with the volunteer’s reasoning, continue to discuss the problem until students reach a consensus. Have each student cross out the volunteer’s choice and write the reason on the line next to the incorrect answer. Ask for a second volunteer to rule out another answer and proceed in the same way. Once two answers are eliminated, ask students to estimate to select the correct answer from the two choices that remain, then carefully record this answer in the response grid. \[ \frac{1}{10} \text{ of 90} \] (B is correct. The answer must be between half and \( \frac{3}{5} \) of 90, so A and D are too great and C is too small.)

Today’s Challenge
Student page 40 Have students work independently on student page 40. Be sure to remind them to explain their reasoning on the lines provided.


Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 40. They should write the correct answers on the back. Have them be careful to write distracters that can be eliminated by estimating. Have them share their problems with a friend, who will estimate (instead of computing) the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 40 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students confidently estimate products of multiplication with decimals?
Math Maze

Week 9 • Activity 41

Materials
Student page 41
Math Maze Cards (Week 9 Activity 41)

Concept and Handbook References
Evaluate expressions involving integers.
(MOC 206, 164)

Background
An algebraic expression is a collection of numbers, variables, operations, and grouping symbols.
Examples:
• \(3x + 12\) three times \(x\) plus twelve
• \(x - 7\) \(x\) minus seven
• \(2(x + 9)\) two times the sum of \(x\) and nine

To evaluate an expression for a specific value of the variable, you replace the variable by the number. Then simplify the numerical expression.
Example:
What is the value of \(3x + 12\) when \(x = -5\)?
Evaluate: \(3x + 12 = 3(-5) + 12\)
\[= -15 + 12\]
\[= -3\]

The order of operations is:
• First, do the operations within grouping symbols, including parentheses, brackets, braces, absolute value symbols, and fraction bars.
• Second, do the multiplication and division left to right.
• Third, do the addition and subtraction left to right.

Get Started
Talk with students about expressions and how to evaluate them. Review rules for operations with integers and then do a few practice examples.
• What is the value of \(-4x - 3\) when \(x = 2\)? (-11)
• What is the value of \(x^2 + 2\) when \(x = -6\)? (38)
• What is the value of \(13 - x\) when \(x = -8\)? (21; remember that to subtract a negative number, add its opposite, so \(13 - (-8)\) is the same as \(13 + 8\))

Remind students about the order of operations and raising a number to a power. In the expression \(4 + 2x\), there are two operations, addition and multiplication. When the variable is replaced by a number, it is multiplied by two before four is added.

Today's Challenge
Distribute the 18 Math Maze cards for week 9. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 188.

Student page 41 When the group has finished playing the game, have students open their books and complete the Today's Challenge activity.

Answers for student page 41: 1. -17 2. 11 3. 31 4. -3 5. 5 6. -1 7. -15 8. -16 9. 60 10. 2

Go Further
Student page 41 Encourage students to look at the results of evaluating an expression for different values of the variable.

Answers for student page 41: 11. 7, 5, 3, 1, -1, -3, -5 12. Check students’ work. The absolute value of each answer is odd. For consecutive values of \(x\), the value of the expression increases by two.

Assessment
Student self-assessment page 41 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students evaluate an expression when the value of the variable is given?
Materials
Student page 42
Colored squares
Scissors
Rulers
Graph paper (optional)

Concepts and Handbook References
Use the Make a List and Draw a Diagram problem-solving strategies. (MOC 480, 483)
Work with translations, rotations, and reflections. (MOC 384–388)
Practice spatial visualization with nets. (MOC 395)

Background
A net is a two-dimensional figure that folds up to form a three-dimensional figure. There are eleven different nets of a cube.

Get Started
Pass out six colored squares, scissors, and graph paper. Draw this net on the board.

Ask students to make an accurate drawing of the same net, modeling first with colored squares. Once students finish their drawings, ask them to cut them out and verify that it will fold to form a cube.

Today's Challenge
Student page 42 Challenge students to find all of the other nets of a cube. Here are hints for students who need help.
- Verify your nets by drawing them on graph paper, then cutting out and folding up each net.
- Organize your search: how many different nets can you make starting with four squares in a row?

Answers for student page 42: 1. These are all of the possible nets of a cube. Students do not need to have found them all until next time.

Look Ahead
Tell students that they will study their nets next time, and will try to be sure that they have all possible nets for a cube by organizing their search.
Materials
Student page 43
Graph paper
Scissors and tape

Concepts and Handbook References
Use the Make an Organized List and Draw a Diagram problem-solving strategies. (MOC 480, 483)
Work with rotations and reflections. (MOC 386, 388)
Practice spatial visualization with nets of cubes. (MOC 395)

Get Started
Pass out scissors, tape, and graph paper to pairs of students.

Go Further
Student page 43 Encourage students to try to be sure that they have all of the different nets of a cube. Remind them to watch for reflections and rotations of the nets they have already found.

Answers for student page 43: Check students’ work. Possible answers are provided.

Things we discovered:
• There can only be one row containing more than two squares.
• There are no nets containing a two by two square.
• There are no nets with five squares in a row.

1–2. Check students’ work.

Assessment
Student self-assessment page 43 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Did students avoid any net duplications because of rotations or reflections? Were they able to offer an argument for only 11 nets of a cube?
Materials
Student page 44
Blank paper (heavyweight if possible) or index cards

Concepts and Handbook References
Review finding percents and fractional amounts. (MOC 161, 442-444)
Use inverse operations. (MOC 224, 479)

Get Started
Review how to find a fraction of a number and a percent of a number and to work backward, using inverse operations, to find the fraction or percent. Use these examples to promote discussion.
• Find $\frac{2}{3}$ of 36 (Think about $\frac{1}{3}$ of 36, which is 12 and double it. So $\frac{2}{3}$ of 36 = 24.)
• Find 75% of 80. (This is equivalent to finding $\frac{3}{4}$ of 80. Think $\frac{1}{4}$ of 80, which is 20 and multiply by three. So 75% of 80 is 60.)
• Find $n$ so that $n\%$ of 48 is 96. (Think if $n\% \times 48 = 96$, then $\frac{n}{100} = \frac{96}{48}$. Since 96 is twice 48, $n$ must be twice 100. $n = 200$.)
• Find a fraction $n$ so that $n \times 28 = 7$. (Think $\frac{7}{28} = \frac{1}{4}$, so $n$ is $\frac{1}{4}$.)

Student page 44 Have students complete the two examples at the top of student page 44. For example 1, $\frac{1}{3}$ of 24 is eight. Now find $n$ so that $n\%$ of 32 is eight. Think: $\frac{8}{32}$ is $\frac{1}{4}$ and $\frac{1}{4}$ is equivalent to 25%. For example 2, 200% of 16 is 32. Now find a fraction $n$ so that $n \times 40 = 32$. Think: $\frac{32}{40} = \frac{4}{5}$.

Today’s Challenge
Student page 44 Have students complete the table on student page 44. Explain that they are to find the missing number so that the value represented in the fraction column equals the value represented in the percent column.

Answers for student page 44: 1. 25  2. 300  
3. $\frac{1}{4}$  4. 25  5. 150  6. $\frac{1}{2}$  7. $\frac{1}{4}$  8. $\frac{1}{3}$  9. $\frac{1}{4}$  10. 66$\frac{2}{3}$%

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 20 small pieces of paper or 20 index cards. Have the students use one slip of paper or card to copy the information from each box in exercises 1–10 on student page 44.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a 4 × 5 array. The first player turns over any two cards. If the cards match (for example, $\frac{1}{3}$ of 24 and 25% of 32), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 44 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students able to use mental math to calculate the percent or fractional amount of a number?
Materials
Student page 45

Concepts and Handbook References
Analyze errors. (MOC 533)
Find circumference and area of a circle.
(MOC 372–373, 375)

Get Started
Draw this circle on the board and ask students to compute both the circumference and the area. Students should leave all their answers in terms of \( \pi \).

\[ C = 12\pi \text{ cm}; \quad A = 36\pi \text{ sq. cm} \]

As part of the discussion, clarify these points.

- The radius is used to compute the area \( A = \pi r^2 \).
- Either the radius \( C = 2\pi r \) or the diameter \( C = \pi d \) may be used to compute the circumference.

Student page 45 Have students analyze the examples at the top of student page 45. Let them work alone for a few minutes, then ask for a volunteer to choose an example and explain why it is correct or incorrect. If students disagree, have the class continue to discuss the problem until they reach a consensus. Continue until all four examples have been analyzed. (B, D, F, and G are correct; the others are incorrect because \( \pi \) does not distinguish between radius and diameter. A. 16\( \pi \) sq. cm
C. 36\( \pi \) sq. in. E. 49\( \pi \) sq. ft H. 12\( \pi \) ft)

Today's Challenge
Student page 45 Have students work independently to solve the problems on student page 45. Ask them to select problems they think Art would solve correctly and those he would do incorrectly, and to explain their thinking. Then, ask them to give some advice to Art about computing area and circumference of circles.

Answers for student page 45: 1. Art would find the area of B and C correctly. A. 64\( \pi \) sq. in.; 16\( \pi \) in. B. 900\( \pi \) sq. cm; 60\( \pi \) cm C. 16\( \pi \) sq. cm; 8\( \pi \) cm D. 81\( \pi \) sq. in.; 18\( \pi \) in. 2. Check students’ work. Advice to Art should mention careful distinction between radius and diameter.

Go Further
On a separate sheet of paper have each student write his or her name and create three problems involving the area of a circle. One should be a problem that Art would answer correctly, and two should be problems that Art would not answer correctly. They should write the correct answers on the back. Have students share their problems with a friend, who is to identify the one problem that Art would solve correctly, solve all the problems, and sign his or her name. If there is disagreement, students should edit their problems.

Assessment
Student self-assessment page 45 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students explain how to compute the area of a circle given the length of either the radius or the diameter?
Materials
Student page 46
Math Maze Cards (Week 10 Activity 46)

Concept and Handbook References
Add fractions. (MOC 104–107)

Background
Numerator $\rightarrow$ \[ \frac{5}{6} = \frac{10}{12} \] $\left\{ \text{ Equivalent} \right\}$
Denominator $\rightarrow$ \[ \frac{3}{4} \]

A common denominator is a common multiple of the original denominators. Often, the least common multiple is used for this purpose.

\[
\frac{5}{6} + \frac{3}{4} = \frac{5 \times 2}{6 \times 2} + \frac{3 \times 3}{4 \times 3} = \frac{10}{12} + \frac{9}{12}
\]

\[
= \frac{19}{12} = \text{Sum of Numerators}
\]

\[
= 1\frac{7}{12}
\]

Get Started
Talk about the role of the numerator and denominator in fractions. Then, discuss the reasons for making sure the denominators are the same before adding. (Adding \(\frac{1}{4}\) to \(\frac{1}{10}\) is like adding a quarter to a dime. To add the coins you need to have the same units, cents.) Pose these problems and ask students to explain their thinking.

• How much is \(\frac{1}{6} + \frac{1}{6}\)? (\(\frac{2}{6}\) or \(\frac{1}{3}\))
  When the denominator is the same, add the numerators.

• How much is \(\frac{2}{3} + 1\frac{1}{3}\)? (\(1\frac{11}{12}\))
  You need to find a common denominator.

A common denominator is 12; another way of expressing this problem is:

\[
\frac{2}{3} \rightarrow \frac{2 \times 4}{3 \times 4} \rightarrow \frac{8}{12}
\]

\[
+ 1\frac{1}{4} \rightarrow 1 + (\frac{1 \times 3}{4 \times 3}) \rightarrow 1\frac{3}{12}
\]

\[
1\frac{11}{12}
\]

Today’s Challenge
Distribute the 18 Math Maze cards for week 10. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have pencil and paper available for today’s Math Maze.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 189.

Student page 46
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity.

Answers for student page 46:
1. \(\frac{11}{12}\) 2. \(\frac{5}{6}\) 3. \(\frac{1}{2}\)
4. \(\frac{5}{6}\) 5. \(\frac{5}{6}\) 6. \(\frac{2}{3}\) 7. \(\frac{1}{3}\) 8. \(\frac{1}{4}\) 9. \(\frac{3}{8}\) 10. \(\frac{2}{5}\) 11. \(\frac{3}{12}\)
12. FRACTION SUM FUN

Go Further
Student page 46
Have students edit exercises 13–14 if their friends do not get the same values for their expressions.

Answers for student page 46: Check students’ work. Possible answers are provided. 13. \(\frac{1}{6} + \frac{5}{6} = 2\)
14. \(\frac{2}{3} + \frac{1}{9} = \frac{2}{2}\)

Assessment
Student self-assessment page 46
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Do students consistently remember when to look for a common denominator?
Materials
Student page 47
Calculators
Chart paper and markers

Concept and Handbook Reference
Identify prime and composite numbers. (MOC 058–060)

Background
French mathematician Marin Mersenne proposed an expression for generating prime numbers: $2^n - 1$. His expression may generate primes only if the value of $n$ is itself a prime number (and even in that case, not always). For the prime number $n = 3$, the expression yields $2^3 - 1 = 7$, also prime. For the composite number $n = 4$, the expression yields $2^4 - 1 = 15$, also a composite number.

The search for prime numbers continues today. Prime numbers are useful for encrypting computer programs and files. Mersenne’s formula is still used to generate prime numbers. Unfortunately, his scheme for generating prime numbers when $n$ is prime fails when $n = 11$. It also fails for much greater values of $n$, but no one can yet predict which values for $n$ fail to produce a prime number until that number is tested. Thus, while Mersenne’s expression cannot guarantee a prime number, it is as good a predictor as any other method yet known.

Get Started
Write 28 and 29 on the board. Ask students how these numbers are different. You will get answers like odd and even, or one is greater, and so forth. Continue the discussion until students recognize that 28 is composite and 29 is prime. Define a prime number as a number whose only factors are itself and one. Composite numbers have more than two factors.

Today’s Challenge
Student page 47 Suggest that students might find calculators useful in their search for the Mersenne primes and also for finding factors for the non-prime numbers that Mersenne’s expression generates. Have them work in pairs or small groups with calculators available.

If students are having a difficult time determining whether a number is prime or composite, you may wish to give these hints.

- Use a calculator to check for potential factors by using the divide key.
- Limit the search for factors to prime factors. Try until you find a factor or until you have tried factors through those close to the square root of the number you are testing.
- Think about using divisibility rules—as soon as one factor pair is found, students have their answer.

### Answers for student page 47: 1.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$2^n - 1$</th>
<th>Prime or Composite?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Neither</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>Prime</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>Prime</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>Composite</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>Prime</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>Composite</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>Prime</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>Composite</td>
</tr>
<tr>
<td>9</td>
<td>511</td>
<td>Composite</td>
</tr>
<tr>
<td>10</td>
<td>1023</td>
<td>Composite</td>
</tr>
</tbody>
</table>

Look Ahead
Copy the table of answers for exercise 1, page 47, on chart paper so students will have accurate data to use next time.
Materials
Student page 48
Calculators

Concepts and Handbook References
Identify prime and composite numbers. (MOC 058–060)
Use exponents to write prime factorization. (MOC 061)
Use the Look for a Pattern and Make a List problem- solving strategies. (MOC 484, 480)

Get Started
Display the table of answers for student page 47 so all students are working with accurate data.

Go Further
Student page 48 Have students work in small groups with access to calculators.

Answers for student page 48: Check students' work. Possible answers are provided.

Things we discovered:
• For \( n = 1-10 \), if \( n \) is prime, \( 2^n - 1 \) is prime.
• For \( n = 1-10 \), if \( n \) is composite, \( 2^n - 1 \) is composite.
• If \( n \) is one, \( 2^n - 1 \) is one.
• \( 2^n - 1 \) gets large very fast.

1. 3, 7, 31, 127. 2. Probable answer: yes 3. For \( n = 11 \), \( 2^n - 1 = 2047 \); 2047 is divisible by 23, so it is not prime. The prediction is not true and the expression may fail again to produce a prime when \( n \) is prime. 4. When \( n \) is not prime, \( 2^n - 1 \) will not generate a prime. If \( n \) is prime, you still must test \( 2^n - 1 \) for factors. 5. 8191 (\( n = 13 \))

Assessment
Student self-assessment page 48 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Did students devise different strategies for determining which Mersenne numbers might be prime?
Materials
Student page 49
Math Jumble activity poster and digit cards
Calculators (optional)

Concept and Handbook References
Use exponents. (MOC 071, 075, 076, 080)

Background
In the expression $2^3 = 8$, two is the base, three is the exponent, and eight is the third power of two.

Get Started
Begin by reviewing the vocabulary associated with powers and exponents. Ask students to evaluate these expressions.
- What is $5^2$? (25)
- What is $8^2$? (64)
- What is $3^3$? (27)

You might also wish to review how to find a power on the calculator using the $y^x$ key. To find $2^3$ on the calculator you got with your kit, use this set of keystrokes.

Today's Challenge
Use the 0–9 digit cards to construct this poster.

<table>
<thead>
<tr>
<th>8</th>
<th>9</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

Explain that the object of today's Math Jumble is to find a number and its second power.

A one-digit number and its square (second power) can be made with any two or three adjoining digits (horizontally, vertically, and/or diagonally) on the poster. For example the two fives and a two are on the right side of columns two and three. Use them to write:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

Numbers on the poster can be used more than once but not in the same problem. Record students' answers on the board by writing the number and its square. Then have students supply the squares of any one-digit numbers not found on the poster.

Possible answers: $8^2 = 64$, $7^2 = 49$, $3^2 = 9$, $5^2 = 25$, $4^2 = 16$, $2^2 = 4$, $9^2 = 81$; Students will not find $6^2 = 36$ or $1^2 = 1$

Student page 49 Have students use the Math Jumble on student page 49 to find numbers and their squares.

Answers for student page 49: 1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
</tr>
</tbody>
</table>

Go Further
Student page 49 Have students answer the questions on the student page.

Answers for student page 49: Check students' work. Possible answers are provided. 2. $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $3^4 = 81$, $4^4 = 256$, $4^3 = 243$ 5. $2^6 = 64$, $3^6 = 729$, $6^3 = 64$

Assessment
Student self-assessment page 49 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students identify powers of one-digit numbers?
Materials
Student page 50

Concepts and Handbook References
Find area. (MOC 356, 366, 375)
Compute with measures. (MOC 357)
Practice the Ignore Unneeded Information problem-solving skill. (MOC 489)

Get Started
Tell students that some problems on tests contain more information than is needed. The extra information can sometimes make the problem more complex and distract them from their task.

Review the formula for area of a triangle. \( A = \frac{1}{2}bh \)

Draw this diagram on the board.

![Diagram of a triangle with sides 12 ft, 13 ft, and 5 ft, and an angle of 67°.]

Ask students to tell you what information is not required if they want to find area. (measure of angle \( B \), length of hypotenuse) What information is not required if they want to find the measure of angle \( A \)? (lengths of sides) What information is not required if they want to find perimeter? (measures of angles)

Student page 50 Work with the class on the example at the top of student page 50. Ask students to first list all unnecessary information and then compute the area of the square. Since it is given that \( ABCD \) is a square, the right-angle indicator and the lengths of all but one side are not needed. Or, the label that says \( ABCD \) is a square and any two adjacent side lengths are not needed. \( A = 25 \text{ sq. in.} \)

Today's Challenge
Student page 50 Tell students that they will highlight important information and/or eliminate unnecessary information and then compute the areas of the figures.

Answers for student page 50:
1. not needed: either length of diagonal and fact that \( FACE \) is a rectangle or length of diagonal, right angle marker, parallel markers, congruent markers; \( A = 60 \text{ sq. ft} \)
2. not needed: either fact that \( WHY \) is a triangle, fact that \( HU \) is an altitude, lengths of sides \( WH \) and \( HY \) or the fact that \( WHY \) is a triangle, right angle marker, lengths of \( WH \) and \( HY \); \( A = 42 \text{ sq. cm} \)
3. not needed: either measures of segments \( EC \), \( BC \), and \( AO \) or \( DO \) and fact that \( AD \) is a diameter or mark at center of circle, lengths of segments \( EC \) and \( BC \); \( A = 100\pi \text{ sq. cm} \)
4. not needed: either facts that \( IRON \) is a parallelogram and \( ID \) is an altitude, length of \( OR \) or congruent markers, parallel markers, right angle marker, and length of \( OR \); \( A = 48 \text{ sq. ft} \)
5. Check students' advice. They should mention paying attention only to information needed to solve the problem.

Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 50. They should write the correct answers on the back. Have them share their problems with a friend, who will cross out unnecessary information, find the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 50 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students distinguish between necessary and extraneous information?
Materials
Student page 51
Math Maze Cards (Week 11 Activity 51)

Concept and Handbook Reference
Add and subtract with absolute values. (MOC 050)

Background
Distance is always a positive value. The absolute value of a number is the distance of that number from zero on the number line regardless of its direction from zero. The absolute value of a number, \( x \), is denoted \( |x| \).

Examples:
\[
|5| = 5
\]
\[
|-4| = 4
\]
\[
|-8 + 6| = |-2| = 2
\]

Note that the absolute value symbols act as grouping symbols for the purposes of order of operations.

Get Started
Review absolute value, and then ask the following questions.

- What is \(|-3 + 5|?\) \((|-3 + 5| \rightarrow |2| = 2)\)
- What is \(|-3| + |5|?\) \((-3| + |5| \rightarrow 3 + 5 = 8)\)
- What is \(|-6 - 10|?\) \((-6 - 10| \rightarrow |-16| = 16)\)
- What is \(|-6| - |10|?\) \((-6| - |10| \rightarrow 6 - 10 = -4)\)

Make sure that students understand the difference between the absolute value of a sum (first example) and the sum of absolute values (second example). Likewise, make sure that students understand the difference between the absolute value of a difference (third example) and the difference of absolute values (fourth example).

Today's Challenge
Distribute the 18 Math Maze cards for week 11. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, "Who has the card with the answer to the question just read?" Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student's card.

The correct sequence of questions and answers is shown on page 190.

Student page 51 When the group has finished playing the game, have students open their books and complete the Today's Challenge activity on page 51 in the student book.

Answers for student page 51: 1. d 2. f 3. g 4. a 5. b 6. e 7. c 8. j 9. i 10. h

Go Further
Student page 51 Have students edit their expressions if their values do not agree with their friends' values.

Answers for student page 51: 11. Check students' work.

Assessment
Student self-assessment page 51 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Are students able to compute sums and differences involving absolute value? Are they aware of the difference between \(|a| + |b|\) and \(|a + b|\)?
Materials
Student page 52
Ruler
Graph paper (optional)
Chart paper and markers

Concepts and Handbook References
Use the Look for a Pattern and Make a List problem-solving strategies. (MOC 484, 480)
Recognize square numbers. (MOC 076)

Get Started
Draw the grid shown here on the board. Ask students how many squares are in the grid.

\[
\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

After students have had a few moments to search for the squares, ask for volunteers to give and explain their answers. Most students should see the nine small, one unit squares. There is one large square, three units per side that fills the entire grid and there are four smaller squares, each two units on a side. It may be easier to talk about squares of various sizes if you label the vertices.

\[
\begin{array}{cccc}
A & B & C & D \\
E & F & G & H \\
I & J & K & L \\
M & N & O & P \\
\end{array}
\]

Today's Challenge
Student page 52 Challenge students to find all of the squares in the grids on the student page. Remind them to keep track of each square they find for next time.

For students who have difficulty finding or recording their squares; provide these hints.
- Copy each figure onto graph paper and use one copy for labeling and counting each square-size.
- Label the vertices or write a letter inside each small square. Use your labels to keep track of the squares you have already counted.
- Look for all squares of a particular size before moving on to other sizes.

Answers for student page 52: Wait until next time to expect all counts to be correct. 1. Figure 1: 1; Figure 2: 5; Figure 3: 14; Figure 4: 30; Figure 5: 55

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
Materials
Student page 53
Graph paper

Concepts and Handbook References
Use the Look for a Pattern and Make a List problem-solving strategies. (MOC 484, 480)
Recognize square numbers. (MOC 076)

Get Started
Fill in the class list of results for student page 52. If there are disagreements, diagram the disputed figure and recount the squares.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
</tr>
</tbody>
</table>

Check students’ work. Sample answers are provided.

Things we discovered:
• The numbers on the diagonals in the table are the same.
• The number of $1 \times 1$ squares is always the square of the Figure number.
• If the dimensions are $n \times n$, then there are $n^2$ $1 \times 1$ squares, $(n - 1)^2 \times 2 \times 2$, $(n - 2)^2 \times 3 \times 3$, and so forth.
• For any $n$, the total number of squares is:
  $n^2 + (n - 1)^2 + (n - 1)^2 + (n - 2)^2 + \ldots + 2^2 + 1^2$.

2. There are 204 squares on a checkerboard:
  $8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1 = 204$

Assessment
Student self-assessment page 53 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students find efficient ways to keep track of their discoveries?

Go Further
Student page 53 Encourage students to be systematic in their search. Offer graph paper if they wish to draw and label diagrams.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Original Dimensions</th>
<th>$1 \times 1$ Squares</th>
<th>$2 \times 2$ Squares</th>
<th>$3 \times 3$ Squares</th>
<th>$4 \times 4$ Squares</th>
<th>$5 \times 5$ Squares</th>
<th>Total Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1 \times 1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$2 \times 2$</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>$3 \times 3$</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>$4 \times 4$</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>$5 \times 5$</td>
<td>25</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>55</td>
</tr>
</tbody>
</table>
Materials
Student page 54
Slips of paper containing fractions

Concept and Handbook References
Find decimal equivalents of benchmark fractions. (MOC 442, 538)

Get Started
Review how to write the decimal form of a fraction. Any fraction can be rewritten with a denominator of 100 and then rewritten as a decimal.

Example: \(\frac{3}{4} = \frac{75}{100} = 0.75\).

A fraction that is equivalent to a repeating decimal can be rewritten by dividing the numerator by the denominator. (\(\frac{1}{6} = 1 \div 6 = 0.16\)) Students should recognize that simplified fractions with denominators of three and six are equivalent to repeating decimals.

Today's Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with one of these fractions written on it: \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{16}\).

You do not need to use all of the fractions, but be sure that one student receives the fraction \(\frac{1}{2}\), since that will be the “Fantastic Finalist.”

Have all students hold their fractions and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.

- If your fraction is equivalent to 0.5, sit down. (\(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\))
- If your fraction is equivalent to a repeating decimal, sit down. (\(\frac{1}{7}, \frac{2}{3}, \frac{4}{9}\))
- If your fraction is equivalent to a decimal with a zero in the hundredths place, sit down. (\(\frac{1}{3}, \frac{2}{5}, \frac{3}{10}, \frac{4}{10}\))
- If your fraction is equivalent to 0.25, sit down. (\(\frac{1}{4}\))

At this point, only the student holding the fraction \(\frac{1}{2}\) should still be standing. That student is the “Fantastic Finalist.” Ask the Fantastic Finalist to name the decimal equivalent of his or her fraction. (0.75)

Go Further
Student page 54 Have students complete the activity on the student page.

Answers for student page 54:

1-7. 

8. 0.5 9. Possible answers: \(\frac{1}{2}, \frac{2}{3}\) 10. Possible answers: \(\frac{1}{3}, \frac{1}{16}\)

Assessment
Student self-assessment page 54 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students recognize fraction and decimal equivalents?
Materials
Student page 55

Concepts and Handbook References
Estimate to eliminate incorrect quotients for division with mixed numbers. (MOC 032–035)
Compute with fractions. (MOC 192)

Background
To divide mixed numbers, first write the divisor and dividend as fractions, then divide as you would any other fractions.

Example:
\[ 4\frac{1}{2} \div 3\frac{1}{3} = \left( \frac{9}{2} + \frac{1}{3} \right) \div \left( \frac{24}{8} + \frac{2}{3} \right) \]
\[ = \frac{9}{2} \div \frac{29}{8} \]
\[ = \frac{9}{2} \times \frac{8}{29} \leftarrow \text{Multiply by reciprocal of divisor.} \]
\[ = \frac{72}{58} \]
\[ = 1\frac{7}{29} \]

Get Started
Student page 55 Ask students to look at the example at the top of student page 55. Ask a volunteer to read the problem aloud. Explain that when you have a test problem with several answer choices, it helps if you can quickly eliminate one or more of the choices. Ask students to select a choice that they are sure is incorrect. Ask for a volunteer to explain his or her thinking. If members of the class do not agree with the volunteer's reasoning, continue to discuss the problem until students reach a consensus. Have each student cross out the volunteer's choice and write the reason on the line next to the incorrect answer. Ask for a second volunteer to rule out another answer choice. Once two answers have been eliminated, ask students to select the correct answer from the two choices that remain. (B is correct; Choices A and C are greater than the number being divided (dividend). If the divisor is greater than one, then the answer (quotient) is always less than the dividend. D is too great, the quotient should be between 8 ÷ 2 and 8 ÷ 3.)

Today's Challenge
Student page 55 Have students work independently on the exercises on student page 55, ruling out at least two answer choices and giving reasons for doing so. Then students may select the correct answer from the two remaining choices.


Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 55. Remind them that at least two answer choices should be easy to eliminate by estimating. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 55 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students comfortable eliminating answer choices without computing the exact answer?
Materials
Student page 56
Math Maze Cards (Week 12 Activity 56)

Concept and Handbook Reference
Solve one- and two-step equations involving integers. (MOC 241)

Get Started
Discuss the use of a variable in an expression or equation. Then, ask the following questions.

• If \(5n = -15\), what is \(n\)? (-3) How do you know?
• If \(10 + n = 16\), what is \(n\)? (6) How do you know?
• If \(4n + 10 = -50\), what is \(n\)? (-15) How do you know?

Help students to think about working backward (using inverse operations) to solve these equations. In the first equation, \(n\) is multiplied by five to get \(-15\). To work backward, divide \(-15\) by five to solve for \(n\). In the third equation, \(n\) is multiplied by four and then 10 is added to get \(-50\). To work backward, reverse the steps by subtracting 10 from \(-50\) to get \(-60\), then dividing by four to get a value for \(n\) of \(-15\). Some students may also think, What number can I multiply by four and then add 10 to get \(-50\)? They should recognize before computing that \(n\) must be negative.

Today’s Challenge
Distribute the 18 Math Maze cards for week 12. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today’s Math Maze.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 190.

Student page 56
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 56 in the student book.

Answers for student page 56:
Check students’ work. Possible operations are provided. 1. 9; add 13
2. 19; subtract 9 3. -6; divide by -9 4. -5; subtract 11 and take the opposite of the difference 5. -72; multiply by 12 6. 4; subtract 8 then divide by 2 7. 11 or -11; take the square root of 121 8. -8; divide by 12 9. 9; add 20 then divide by 5 10. 11; subtract 35 then divide by 10

Go Further
Student page 56
Have students write their own equations.

Answers for student page 56: 11–15. Check students’ work.

Assessment
Student self-assessment page 56
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Can students work backward to find the value of the variable in an equation?
Materials
Student page 57
Chart paper and markers

Concepts and Handbook References
Find multiples. (MOC 067)
Use the Look for a Pattern problem-solving strategy. (MOC 484)

Background
The digit root of a number is found by reducing the number to a single digit by finding the sum of all its digits. For example the digits of 174 have a sum of 12 \((1 + 7 + 4 = 12)\). The digits of 12 have a sum of three \((1 + 2 = 3)\). The digit root of 174 is three.

Get Started
Ask students if they have ever heard of a digit root. Be sure that they do not confuse digit root with square root. If any student has heard of a digit root, ask him or her to demonstrate how to find the digit root of 174. If no students have heard of a digit root, then work through the examples on student page 57 with the class.

Today's Challenge
Student page 57 Encourage students to look for patterns as they work.

Answers for student page 57: 1.

<table>
<thead>
<tr>
<th>Base Number</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
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<table>
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<th>9</th>
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<td>72</td>
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<tr>
<td>54</td>
<td>54</td>
<td>63</td>
<td>72</td>
<td>81</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
</tr>
</tbody>
</table>

Look Ahead
On chart paper, make a copy of the tables on student page 57.
Materials
Student page 58

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Use divisibility tests. (MOC 069)

Get Started
Have students fill in the enlarged copy of the table from student page 57 that you prepared last time. If there are any disagreements, be sure that all of the answers are correct before proceeding with this activity.

Go Further
Student page 58 Have students work in pairs to look for patterns in digit root cycles.

Answers for student page 58: Check students’ work. Possible answers are provided.

Things we discovered:
• Three and six have the same three digits in their digit root cycle (3, 6, and 9 in different orders).
• Nine has a single digit cycle (9).
• The digit root cycle of eight lists the counting numbers 1–9 in reverse order, starting with eight and continuing to one, then starting with nine and counting down.
• The digit root cycles for two and seven are nearly mirror images of each other.
• The digit root cycle for two counts the even numbers and then odd numbers in order.
• For seven the digit root cycle counts down from seven with odd numbers, and then continues counting down all the even numbers, then odd numbers again.
• The digit root cycles for five and four alternate two intermixed number cycles. For example, the cycle for four is 4 8 3 7 2 6 1 5 9 4.

1. See above. For example, the digit root cycles of three and six are the same numbers: 3, 6, and 9.
2. The digit root cycle for three is 3, 6, 9. You can say that if any number has a digit root that may be divided by three, then the number can be divided by three. If the digit root of any number is nine, then the number must be divisible by nine.

Assessment
Student self-assessment page 58 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students persist in the search for interesting patterns?
Materials
Student page 59

Concept and Handbook References
Write fractions as repeating decimals.
(MOC 044, 023)

Get Started
The decimal equivalents of simplified fractions with denominators of three, six, and nine are repeating decimals. Demonstrate by rewriting \( \frac{1}{6} \) as a decimal.

\[
\begin{align*}
0.16 & \quad \text{The bar over a digit (or group of digits) indicates the part of the quotient that repeats.} \\
6 \overline{1.000} & \\
- 6 & \\
40 & \\
36 & \\
40 & \quad \text{When your remainder repeats, you know your quotient will repeat.}
\end{align*}
\]

Students should note that \( \frac{1}{6} \) is half of \( \frac{1}{3} \), so 0.16 is half of 0.3.

Student page 59 Have students complete the table at the top of student page 59.

Answers for student page 59:

1. | Word Name | Fraction Form | Decimal Form |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>one ninth</td>
<td>( \frac{1}{9} )</td>
<td>0.1</td>
</tr>
<tr>
<td>one sixth</td>
<td>( \frac{1}{6} )</td>
<td>0.16</td>
</tr>
<tr>
<td>two ninths</td>
<td>( \frac{2}{9} )</td>
<td>0.2</td>
</tr>
<tr>
<td>one third</td>
<td>( \frac{1}{3} )</td>
<td>0.3</td>
</tr>
<tr>
<td>four ninths</td>
<td>( \frac{4}{9} )</td>
<td>0.4</td>
</tr>
<tr>
<td>two thirds</td>
<td>( \frac{2}{3} )</td>
<td>0.6</td>
</tr>
<tr>
<td>five sixths</td>
<td>( \frac{5}{6} )</td>
<td>0.83</td>
</tr>
<tr>
<td>seven ninths</td>
<td>( \frac{7}{9} )</td>
<td>0.7</td>
</tr>
<tr>
<td>eight ninths</td>
<td>( \frac{8}{9} )</td>
<td>0.8</td>
</tr>
</tbody>
</table>

As you ask each question, have students look at their numbers and answer the question. Yes answers will score points.

1. Does your fraction have a numerator that is not one? If yes, score 10 points.
2. Is your decimal less than 0.5? If yes, score 5 points.
3. Does your decimal have a repeating three or six? If yes, score 9 points.
4. Does your decimal have a non-repeating digit in the hundredths place? If yes, score 8 points.
5. Do your numerator and denominator differ by five? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student write his or her equivalent numbers on the board and explain how the points were scored. Discuss why there can be no zero-point scores for this game. (Everyone will score on either question 1 or question 2.)

Go Further
Student page 59 Have students solve the riddles and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 59: 2. \( \frac{1}{6} \) 3. \( \frac{2}{3} \) 4. Check students’ riddles.

Assessment
Student self-assessment page 59 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students write the repeating decimal forms of common fractions?
Today's Challenge
Student page 60 Have students work in groups of two or three, sharing a calculator. Students can consult with each other, but each student should complete student page 60.

Sample answers for student page 60: 1. Check numbers greater than one.
- $3 \div 6 = \frac{1}{2}$ True
- $6 \div 3 = 2$ True
- $6 \div 2 = 3$ False
- $6 \div 1.5 = 4$ False

Check zero.
- $6 \div 0$ is undefined False
- $0 \div 6 = 0$ False

Check numbers between zero and one.
- $0 \div 6 = 12$ False
- $\frac{1}{2} \div \frac{1}{2} = \frac{1}{2}$ False

Check negative numbers
- $6 \div -2 = -3$ True
- $6 \div -3 = 2$ False
- $-6 \div -2 = 3$ False
- $-6 \div 2 = -3$ False

Write a clear, complete answer to the question.
Pete is not right. For $x + y$,
- if $x$ and $y$ are $> 1$, the statement is true if $y > x$, but may or may not be true if $x > y$.
- if $x$ or $y = 0$, the statement is false.
- if $x$ and $y$ are both negative, the statement is false.

2. Check numbers greater than one.
- $4 \times 4 = 16$ True
- $1.5 \times 1.5 = 2.25$ True

Check zero.
- $0 \times 0 = 0$ False

Check numbers between zero and one.
- $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ False

Check negative numbers
- $-4 \times -4 = 16$ True
- $-1.5 \times -1.5 = 2.25$ True

Write a clear, complete answer to the question.
Arianna is not right. For zero and numbers between zero and one, the statement is false.

3. Advice should include trying different combinations of number types.

Go Further
On a separate sheet of paper, have each student write his or her name and create one problem similar to those on student page 60. They should write the main points of a constructed response and the correct answer on the back. Have them share their problem with a friend, who will write a constructed response, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 60 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students construct a well-reasoned response for a multi-step problem?
Materials
Student page 61
Math Maze Cards (Week 13 Activity 61)

Concepts and Handbook References
Write equivalent fractions, decimals, and percents. (MOC 024, 026, 043–044)
Order fractions, decimals, and percents. (MOC 020, 026, 041)

Background
Some fractions cannot be written as a decimal with a finite number of places. These are repeating decimals. To write such a fraction as a decimal, start out by dividing the denominator into the numerator.

Example:
\[
\frac{1}{3} \rightarrow 0.33 \\
\frac{3}{100} \\
-2 \\
10 \\
-9 \\
1 \leftrightarrow \text{When you see the remainder repeat, you know you have a repeating decimal.}
\]

There are two ways to write this quotient: 0.33\(\frac{1}{3}\) or 0.\(\overline{3}\).

Get Started
Discuss equivalent forms of the same number. Ask students to rewrite fractions with 100 as the denominator (\(\frac{2}{5} = \frac{25}{100}\)). Then ask them to write the fraction equivalent to a decimal number (0.4 = \(\frac{4}{10} = \frac{40}{100}\)). These numbers could now be compared, or written as a percent. Remind students that percent means per hundred.

Examples:
- What is \(\frac{2}{5}\) as a percent? (\(\frac{2}{5} = 0.66\overline{2} = 66\frac{2}{3}\%\))
- What is 2 as a percent? (2 = \(\frac{200}{100} = 200\%\))
- What is 0.4 as a percent? (0.4 = \(\frac{40}{100} = 40\%\))
- What is \(\frac{1}{6}\) as a percent? (\(\frac{1}{6} = 0.16\overline{6} = 16\frac{2}{3}\%\))

Today’s Challenge
Distribute the 18 Math Maze cards for week 13. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 191.

Student page 61 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 61 in the student book.

Answers for student page 61: 1. 0.6, 66\(\frac{2}{3}\)% 2. 1\(\frac{1}{4}\), 25% 3. 0.01, 1% 4. \(\frac{1}{20}\), 5% 5. \(\frac{4}{5}\), 80% 6. 0.125, 12.5% 7. \(\frac{1}{3}\), 33\(\frac{1}{3}\)% 8. 0.6, 60% 9. \(\frac{19}{20}\), 95% 10. 0.55, 55%

Go Further
Student page 61 Have students place numbers on the number line.

Answers for student page 61: 11.

Assessment
Student self-assessment page 61 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students have a good sense of the percent equivalents of common decimals and fractions?
Materials
Colored squares
Student page 62
Chart paper and markers

Concept and Handbook Reference
Use the Use Logical Reasoning problem-solving strategy. (MOC 492)

Get Started
Explain that today students will be collecting data about the number of trips it takes to cross a river given certain restrictions. One of the restrictions is that the boat used for crossing the river can hold at most one adult or two children. It cannot hold two adults or an adult and a child. The first thing they will need to do is to figure how to get everyone over to the other side.

Today’s Challenge
Student page 62 Have students work in pairs to figure out how to get everyone across the river. Suggest that they draw a river on a sheet of paper and use two different colors of colored squares as people to simulate the solution.

Answers for student page 62: 1. Have two children cross in the boat and have one child stay on that side and the other child bring the boat back so an adult can cross. The second child then brings the boat back across. 2. Check students’ work. It will take 17 trips.

3.

<table>
<thead>
<tr>
<th>Number of Adults</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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<th>8</th>
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</thead>
<tbody>
<tr>
<td>Number of Trips</td>
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<td>9</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>25</td>
<td>29</td>
<td>33</td>
<td>37</td>
<td>41</td>
<td>45</td>
</tr>
</tbody>
</table>

4.

<table>
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<tr>
<th>Number of Children</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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<td>11</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
</tr>
</tbody>
</table>

Look Ahead
On chart paper, copy the two tables on student page 62.
Materials
Student page 63

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Write an equation to describe a functional relationship. (MOC 205, 244)
Solve multi-step equations. (MOC 238–242)

Get Started
Work with the class to fill in a large chart containing the tables from student page 62 (see answers for student page 62). If there is any disagreement, draw a diagram on the board and simulate the problem.

Go Further
Student page 63 Have the students work in pairs for this activity.

Answers for student page 63: Check students’ work. Possible answers are provided.

Things we discovered:
• Every time you add an adult the number of trips goes up by four.
• It takes four trips to get an adult across the river and have both children return to the other side.
• It only takes one trip for two children to cross the river.
• When \( c > 2 \), it takes two trips per child, except for the last two children.
• There is never an even number of trips to get to the opposite side of the river.
• The number of trips needed to transport \( a \) adults and two children is one more than a multiple of four.

1. \( t = 4a + 1 \) 2. 109 3. 14 4. 63 trips is not possible, since the number of trips is always one more than a multiple of four. 5. For \( c > 1 \), \( t = 2c - 3 \)
6. For \( c > 1 \), \( t = 4a + 2c - 3 \) 7. 63

Assessment
Student self-assessment page 63 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students evaluate an equation by substituting in values for the variables?
Materials
Student page 64
Blank paper (heavyweight if possible) or index cards

Concepts and Handbook References
Evaluate algebraic expressions involving negative numbers. (MOC 108, 136, 164, 193)
Use order of operations. (MOC 207–209)

Background
See page 41 for a review of algebraic expressions and order of operations.

Get Started
Talk with students about expressions and how to evaluate them, then do a few practice examples.
• What is the value of $4x - 3$ when $x = -2$? (-11)
• What is the value of $-5(x + 2)$ when $x = 3$? (-25)
• What is the value of $\frac{1}{2}x - 4$ when $x = 12$? (0)
• What is the value of $4 - 2x$ when $x = 5$? (-6)

Remind students about the order of operations. In the expression $4 - 2x$, there are two operations involved, addition and multiplication. If $x = 5$, then five is multiplied by two before the product (10) is subtracted from four.

Today's Challenge
Student page 64: Have students complete the table on page 64 of their books. In each case, students are to write the value of the expression when $x$ is replaced by its assigned value.


Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 20 small pieces of paper or 20 index cards. Have the students use one slip of paper or card to copy the information from the left side of each box in the table on student page 64. Students will not copy the answers from the right side of each box.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a $4 \times 5$ array. The first player turns over any two cards. If the cards match (have the same value when the expressions are evaluated), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 64 Have students circle one of the two choices to describe how they feel about this activity.
Assessment tip Can students evaluate simple algebraic expressions using mental math?
Materials
Student page 65

Concepts and Handbook References
Add integers. (MOC 108)
Use absolute value to determine the sign of a sum. (MOC 050, 108)
Use the Use Logical Reasoning problem-solving strategy. (MOC 492)

Get Started
Ask students the absolute value of $-6$, denoted as $|{-6}|$. (6) Ask students the absolute value of $9$, denoted as $|9|$. (9) Remind them that the absolute value of a number is its distance from zero regardless of the direction. Ask students to evaluate these expressions.

- $14 + 27 (41)$
- $-26 + -38 (-64)$
- $-42 + 18 (-24)$
- $53 + -19 (34)$

Ask students to compose a set of rules for adding signed numbers. Probe for these points.

- When adding two numbers with like signs, add the absolute values of the two numbers; the sum has the same sign.
- If the signs are unlike, subtract the lesser absolute value from the greater absolute value; the sum has the sign of the number with the greater absolute value.
- When the signs are unlike, the sum is between the values of the addends.

Ask them to choose, without computing, the correct value of this expression, and explain why they know what the value is.

$-52 + 28$
A. $-80$
B. $24$
C. $-24$
D. $80$

Choice C is correct because the answer has to be (1) between $-52$ and $28$ since the signs are opposite and (2) negative because the addend with greater absolute value is negative.

Student page 65 Have students work independently on the example at the top of student page 65 and then discuss their answers with them. (A, signs are opposite, so the sum is between $-45$ and $71$; the positive number has the greater absolute value, so the sum is positive.)

Today's Challenge
Student page 65 Have students work independently on student page 65.


Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 65. They will need to pay careful attention to choosing incorrect answer choices. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 65 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students pay careful attention to signs as they use logical reasoning to choose correct sums?
Materials
Student page 66
Math Maze Cards (Week 14 Activity 66)

Concepts and Handbook References
Identify and recognize prime and composite numbers. (MOC 058–060)
Use number theory vocabulary. (MOC 053)

Background
- A prime number has exactly two factors: itself and one. There are 25 primes between one and 100.
- Composite numbers have more than two factors. One is neither prime nor composite because it has only one factor.
- A perfect square is the product of a whole number used as a factor twice. Perfect squares are the set \( \{1, 4, 9, 16, 25, \ldots n^2\} \).
- A perfect cube is the product of a whole number used as a factor three times. Perfect cubes are the set \( \{1, 8, 27, 64, 125, \ldots n^3\} \).

Get Started
Talk about prime numbers and ways to identify them. Ask students to decide whether these numbers are prime. If not, have them name at least one factor other than one and the number itself.
- 7 (yes)
- 9 (no; 3)
- 12 (no; 2, 3, 4, or 6)
- 31 (yes)
- 99 (no; 3, 9, 11, or 33)

Today's Challenge
Distribute the 18 Math Maze cards for week 14. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today's Math Maze.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 192.

Student page 66
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 66 in the student book.

Answers for student page 66: 1. 64 2. 91 3. 97 4. 72 5. 8 6. 15 7. 36 8. 13 9. 23 10. 78

Go Further
Student page 66
Have students think about strategies for finding a least common multiple as they complete these problems.

Answers for student page 66: 11. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39 12. 4, 8, 12, 16, 20, 24, 28, 32, 36 13. 12, 24, 36; these composite numbers are the multiples of 12 less than 40 14. 17, 31, 71

Assessment
Student self-assessment page 66
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Are students able to check quickly to decide whether a number is prime?
Materials
Student page 67

Concepts and Handbook References
Graph in the coordinate plane. (MOC 247–250)
Solve simple equations. (MOC 239–241)
Check solution sets by substitution. (MOC 242)
Compute with positive and negative numbers. (MOC 108, 136, 164, 193)

Background
In a coordinate plane, the horizontal axis plots values of x and the vertical axis plots values of y. These axes divide the plane into four parts, called quadrants.

A table of values (T-table) shows the values of y for selected values of x in a given equation.

Example: \( y = 2x + 3 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

These values of x and y can be used as an ordered pair of coordinates \((x, y)\) for points on a graph for the equation.

Get Started
Draw a coordinate plane on the board. Ask for volunteers to plot these points: \((2, -3)\), \((5, 1.5)\), \((-4, -2)\), \((-5, 0)\), \((0, 3)\) and \((-2.5, 5)\).

Look Ahead
Tell students that next time they will look for clues in the equations that will let them predict how graphed lines will look.

Write the equation \( y = x + 6 \) on the board and draw this table of values without the y-values. Have students fill in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>-5</td>
<td>1</td>
</tr>
</tbody>
</table>

Today’s Challenge
Student page 67 Have students graph the equations on the same axis. Be sure that they create a table of values showing at least two positive numbers, two negative numbers, and zero as x-values.

Answers for student page 67: 1–5.
Materials
Student page 68

Concept and Handbook References
Use the Look for a Pattern and Make and Use a Graph problem-solving strategies to generalize about the graphs of linear equations.
(MOC 245–250, 484, 485)

Background
A coefficient is the numerical factor in an expression like 4a or 25xy. In expressions like x or n + 4, a coefficient of one is implied.

A constant is a term in an expression that will not change.

Example: In 6n + 4, 6n and 4 are terms; 6n is a variable term whose coefficient is 6; 4 is a constant term.

The value of y at the point where a line crosses the x-axis is called the x-intercept. The value of x where a line crosses the y-axis is called the y-intercept.

Get Started
Review vocabulary words (term, coefficient, constant, variable) that describe parts of an expression.

Go Further
Student page 68 Have students work with partners to study the relationships between equations and their graphs.

Answers for student page 68: Check students’ work. Possible answers are provided.

Things we discovered:
- All the equations are of the form y = x + a, where a is some real number.
- The coefficient of x is one.
- The only thing that changes in each expression is the constant term.
- The graph of each equation is a straight line.
- All the lines are parallel to one another.
- The line y = x goes through the origin and bisects the first and third quadrants.

1. The line would be parallel to the others and pass through (0, 7). 2. The line would be parallel to the others and pass through (0, −3). 3. y = x + 3 4. where it crosses the y-axis

Assessment
Student self-assessment page 68 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students evaluate and graph equations by substituting values for variables?
Materials
Student page 69
Math Jumble activity poster and digit cards

Concept and Handbook References
Use order of operations with integers. (MOC 108, 136, 164, 193, 209)

Background
See page 41 for a review of order of operations.

Get Started
Ask students to find the value of the expression
8 + -4 \times 3. If students perform the operations left
to right, they will get a value of 12. If they use the
correct order of operations and perform the multi-
plication first, they will get a value of -4. Review
the order of operations with students and then try it
on these examples.
• 3 + -8 \times 2 - 4 (-17)
• -9 ÷ 3 \times -7 + 8 (29)
• 12 - 4 + -8 ÷ 2 (4)
• -8 \times 5 ÷ -2 + 7 (27)

Today’s Challenge
Use the 0–9 digit cards to construct this poster.

Possible expressions: 6 ÷ 3 \times -2 + 7 = 3;
7 - 4 + -6 \times 8 = -45; 3 + 8 ÷ 4 \times -7 = -11

Student page 69 Have students use the Math
Jumble on student page 69 to write expressions
using three different operations. Numbers in the
expression are written in order from one row or col-
umn; at least one of the numbers must be negative.

Answers for student page 69: 1–4. Check students’
work. Possible answers are provided.
4 + 3 \times -7 - 8 = -25; -3 + 0 \times 6 \div 3 = -3

Go Further
Student page 69 Have students use the grid on the
student page to create a Math Jumble to share with
a friend.

Answers for student page 69: 5–7. Check students’
work.

Assessment
Student self-assessment page 69 Have students cir-
cle one of the two choices to describe how they feel
about this activity.

Assessment tip Can students use order of opera-
tions to correctly evaluate an expression involving
integers?
Materials
Student page 70
Chart or transparency of blank fill-in grid (save for future activities)

Concepts and Handbook References
Use divisibility rules as a way to eliminate distracters. (MOC 069, 528)
Use a machine-scoring style fill-in grid. (MOC 532)

Background
There are a number of ways to check for divisibility by a given number. Here are divisibility rules that should be easy for your students to understand and remember.
• If the digit in the ones place is even, the number is divisible by two.
• If the sum of the digits in a number is divisible by three, the number itself is divisible by three.
• If the number formed by the last two digits of a number is divisible by four, the number itself is divisible by four.
• If the digit in the ones place is zero or five, the number is divisible by five.
• If the number is divisible by two and by three, then it is divisible by six.
• If the sum of the digits in a number is divisible by nine, the number itself is divisible by nine.
• If the digit in the ones place is zero, the number is divisible by ten.

Composite numbers such as 20 follow the rules of divisibility of their factors; that is, numbers divisible by four and by five are divisible by 20. However, numbers divisible by three and by nine are not necessarily divisible by 27. If the pair of factors do not have a common factor, then this rule will work.

Get Started
Display the blank fill-in grid and demonstrate its use. Students need to write their answer in the top row and then fill in the digit, decimal point, or fraction bar that matches what they have written at the top of the column.

Discuss how it may be more efficient to eliminate wrong answer choices than to spend time comput-

ing on a timed test. If you can eliminate wrong answer choices, then you do understand the concept being tested.

If necessary, review divisibility rules, then write these numbers on the board: 21,570; 2530; 3442; and 7560. Ask the class which are divisible by six but not by four. Tell them not to compute, but instead to use divisibility rules. (21,570 is divisible by six but not by four; 2530 and 3442 are not divisible by six, since the sums of their digits are not divisible by three; 7560 is correct; it is divisible by four because the number formed by its ones and tens digits is divisible by four and by six because it is even and the sum of its digits is a multiple of three)

Student page 70 Work with the students on the example at the top of student page 70. (C is correct as 2360 is divisible by both four and five (so 20) but not by three (so not 15); A and B are not divisible by four (so not 20); D is divisible by four and five (so 20), and also by three, (so 15).

Today's Challenge
Student page 70 Have students work independently to solve the problems on page 70. Be sure to remind them to explain their reasoning.


Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on page 70. They should write the correct answers on the back. Have them share their problems with a friend, who will reason out the correct answers, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 70 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students combine divisibility rules for numbers they know, like two, three, four, five, and nine and use them for composite numbers like 15, 18, or 20?
**Materials**
Student page 71
Math Maze Cards (Week 15 Activity 71)

**Concept and Handbook Reference**
Use the Distributive Property. (MOC 220)

**Background**
The Distributive Property states that if \(a\), \(b\), and \(c\) are any three real numbers, then
\[
a(b + c) = a \times b + a \times c.
\]
The Distributive Property holds for subtraction as well. This form of the property says
\[
a(b - c) = a \times b - a \times c.
\]
Multiplication is implied in any term of an equation when \(a\) is placed next to the parentheses, so
\[
a(x + 2) = a \times (x + 2).
\]

**Get Started**
Review the distributive property with students. Be sure to work several examples where a negative integer is the factor and the original expression involves subtraction.

\[
-2(3x - 8) = -2(3x) - (-2)(8) = -6x + 16
\]
Try these problems with students.

- Expand \(4(x + 5)\). \((4x + 20)\)
- Expand \(-2(3x + 7)\). \((-6x - 14)\)
- Expand \(\frac{1}{2}(6x - 14)\). \((3x - 7)\)
- Expand \(-5(4 - 2x)\).  (Accept \(-20 + 10x\) but explain that standard practice is to place the variable term first: \(10x - 20\).)

**Today's Challenge**
Distribute the 18 Math Maze cards for week 15. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

**Instructions for playing Math Maze** Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 193.

**Student page 71**
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 71 in the student book.

**Answers for student page 71**
1. \(8x - 12\) 2. \(5x + 2\) 3. \(-3x + 6\) 4. \(4x - 8\) 5. \(6x + 18\) 6. \(x - 2\) or \(-2 + x\) 7. \(2x + 7\) 8. \(-20x + 12\) 9. \(15x + 10\) 10. \(15x + 10\)

**Go Further**
**Student page 71** Discuss the right-hand style of the Distributive Property where the constant factor is written to the right of the parentheses. Have students complete the remaining problems.

**Answers for student page 71**
11. \(15x - 12\) 12. \(x + 4\) 13. \(-6x + 20\) 14. \(3x + 18\)

**Assessment**
**Student self-assessment page 71** Have students circle one of the two choices to describe how they feel about this activity.

**Assessment tip** Are students able to use the Distributive Property to expand an expression?
Materials
Student page 72
Ruler
Overhead projector and transparent grid (optional)

Concepts and Handbook References
Graph linear equations on the coordinate plane.
(MOC 247–250)
Solve multi-step equations. (MOC 239–241)
Compute with positive and negative numbers.
(MOC 108, 136, 164, 193)

Background
Graphs of equations in the form \( y = ax + b \) are
lines, so the equations are called linear equations.
Equations with the same coefficient of \( x \) produce
graphs of parallel lines. Refer to page 67 for a
review of graphing linear equations.

Get Started
Write the equation \( y = -2x + 6 \) on the board.
Make a table of values and ask students to evaluate
the equation for \( x = 4, -2, 0, 5, \) and \( 3 \). Draw a
coordinate plane on the board or use the overhead
projector. Ask volunteers to graph the points in the
table of values. \([4, -2), (-2, 10), (0, 6), (5, -4),\nand (3, 0)]\) Connect those points with a line.

Today's Challenge
Student page 72 Have students graph the equations
given and label each line with its equation.

Look Ahead
Tell students that next time they will look for ways
to predict how graphs of equations will look by
comparing what they have done today with what
they did for activities 67–68.
Materials
Student page 73

Concept and Handbook References
Use the Look for a Pattern and Make and Use a Graph problem-solving strategies to generalize about graphed equations. (MOC 247-250, 484, 485)

Get Started
If students completed activities 67-68, make sure they are available for comparison to the graphs done for activity 72.

Go Further
Student page 73 Have students work in pairs to look for clues to how the form of an equation predicts the form of its graph.

Answers for student page 73: Check students' work. Possible answers are provided.

Things we discovered:
• All the equations are of the form $y = ax + b$, where $a$ and $b$ are real numbers.
• When the coefficient of $x$ is negative, the line slants from the upper left to the lower right.
• When the coefficient of $x$ is positive, the line slants from the lower left to the upper right.

• The graph of each equation is a line.
• The constant term tells where the graph crosses the $y$-axis.
• The greater the absolute value of coefficient of $x$ the steeper the line.

1. It would be fairly steep and it would intersect the $y$-axis at (0, 2); it would slant up from lower left to upper right. 2. This line bisects the second and fourth quadrants; it goes through the origin.
3. The line slants from upper left to lower right and intersects the $y$-axis at (0, 5). 4. The two lines are parallel, fairly steep, with one passing through (0, -3) and the other through (0, 1). 5. They both slant from upper left to lower right. The one passing through (0, 6) is fairly steep; the other passes through (0, -2) and is less steep. 6 A. $y = 5x - 2$ B. $y = -6x - 1$

Assessment
Student self-assessment page 73 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students see how coefficients and constant terms can allow them to predict the way a linear equation will graph?
Materials
Student page 74
Slips of paper containing multiples of 12

Concepts and Handbook References
Identify patterns in multiples of 12. (MOC 484)
Solve for linear dimensions in area, perimeter, and circumference formulas. (MOC 346, 356, 366, 372)

Background
- Area of a rectangle = \( \ell \times w \), where \( \ell = \) length and \( w = \) width
- Area of a triangle = \( \frac{1}{2}bh \), where \( b = \) base and \( h = \) height
- Perimeter of a polygon = sum of side lengths
- Circumference of a circle = \( 2\pi r \), where \( r = \) radius

Go Further
Student page 74 Have students complete the activity on the student page.

Answers for student page 74:

1–4.

5. 76 6. Circle 120, 1008, 432, 900, 864
7. Possible answer: \( P = 2\ell + 2w \);
\( 60 = 2 \times 18 + 2w, w = 12 \), so the multiples of 12 are circled; the number must be divisible by both three and four.

Assessment
Student self-assessment page 74 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students identify multiples of 12? Can they solve for the linear dimensions of a plane figure, given sufficient information?
Materials
Student page 75
Calculators

Concepts and Handbook References
Read and interpret stem-and-leaf plots. (MOC 303)
Find the mean, median, mode, and range for a data set. (MOC 272–276)
Take special care when analyzing large data sets. (MOC 100, 114)

Background
A stem-and-leaf plot is a form of bar graph in which numerical data are plotted using the actual numbers. If the data are a set of two-digit numbers, then the tens digits serve as the stems and the ones digits are the leaves.

Mean \[ \text{Sum of numbers} \div \text{number of numbers} \]
Median Middle number or mean of middle two numbers for data arranged in order
Mode Most frequent number
Range Difference between greatest and least numbers

Get Started
If necessary, review the terms mean, median, mode, and range. Put this stem-and-leaf plot on the board. It is a plot of the age at inauguration of the 25 U.S. Presidents before President George W. Bush.

25 Presidents' Ages at Inauguration

| 4 | 2 3 6 6 7 9 |
| 5 | 0 1 1 2 4 4 5 5 5 6 6 |
| 6 | 0 1 2 4 9 |

Ask students to read the definition of stem-and-leaf plot on student page 75, then study the plot on the board and suggest what can be learned from it. Probe for these points.

- The youngest president was 42 years old.
- The oldest president was 69 years old.
- The median age for the last 25 presidents was 54 years.
- The range of ages of the presidents was 27 years.
- The most frequent inaugural age (mode) was 55 years.
- The mean age for the last 25 presidents was 53.52 years. (At this point, emphasize how important it is to double-check a calculator sum because it is very easy to mis-key when entering so many addends. A good way to double-check a sum is to subtract each addend from the sum. If you run out of addends at the same time your calculator display reads zero, you probably added correctly.)

Student page 75 Give students about five minutes to study the plot at the top of student page 75 and answer the questions. Then discuss their answers to be sure everyone understands. (A. 15 B. Well more than half of the lectures had fewer than 40 people; almost half of the lectures had 30–39 people.)

Today's Challenge
Student page 75 Have students sit in small groups with access to calculators, but encourage them to complete the page on their own.

Answers for student page 75: 1. C 2. B 3. A 4. D 5. C 6. Check students' advice. They might mention not overlooking None of these as a possible answer, but double-checking your work before recording that as an answer.

Go Further
On a separate sheet of paper, have each student write his or her name and create a stem-and-leaf plot and some related questions similar to those on student page 75. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 75 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students know how to read a stem-and-leaf plot?
Materials
Student page 76
Math Maze Cards (Week 16 Activity 76)

Concepts and Handbook References
Write equivalent forms of decimals, fractions, and percents. (MOC 024, 026, 043–044)
Order fractions, decimals, and percents. (MOC 049)

Background
See page 61 for a review of the division method for writing fractions as decimals.

A common error involves percent forms of repeating decimals.

Examples:
• \[ \frac{4}{3} = 1\frac{1}{3} = 133\frac{1}{3}\% \]
• \[ 1\frac{1}{3}\% = \frac{4}{300} = \frac{4}{100} = 0.013 \]

Get Started
Work with students to make a table of decimal equivalents for thirds and eighths, using the division method. Post the equivalents where students can see them.

Today’s Challenge
Distribute the 18 Math Maze cards for week 16. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Do not provide paper and pencil. If students get stuck, they may refer to the table of equivalents you posted earlier.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 194.

Student page 76 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 76 in the student book.

Answers for student page 76: 1. \( \frac{3}{100} \); 0.03 2. \( \frac{4}{5} \); 80% 3. 0.625; 62.5% 4. \( \frac{3}{5} \); 0.6 5. \( \frac{5}{20} \); 15% 6. 1.2; 120% 7. \( \frac{3}{5} \) or \( 1\frac{1}{2} \); 1.5 8. 0.9; 90% 9. \( \frac{11}{10} \) or \( 1\frac{1}{10} \); 110% 10. \( \frac{3}{2} \); 0.2

Go Further
Student page 76 Encourage students to use benchmarks of 0, \( \frac{1}{2} \), and 1 to order these numbers.

Answers for student page 76: 11. 3%, 0.08, \( \frac{9}{100} \), 0.125, 0.65, \( \frac{3}{5} \), 80%, 120%, \( \frac{5}{4} \)

Assessment
Student self-assessment page 76 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students have a good sense of equivalent forms of common decimals, fractions, and percents?
Materials
Student page 77
Overhead projector and transparency containing various line segments and angles (optional)
Protractors
Centimeter rulers
Calculators

Concepts and Handbook References
Study patterns in similar right triangles. (MOC 376)
Find ratios. (MOC 422–427)

Background
In similar figures corresponding angles are congruent, and corresponding sides are in proportion. The fact that their corresponding sides are in proportion leads to a branch of mathematics called trigonometry. The functions of the sine, cosine, and tangent are ratios of sides in right triangles. Since all right triangles with congruent angles are similar, the ratios associated with the angles are constant no matter what the size of the triangle.

Get Started
Pass out centimeter rulers and protractors and ask students to practice measuring short line segments in millimeters and angles in degrees. Ask several students to measure the same angle (on an overhead projector if you have one). Be sure that students understand that there are always errors in measurement because tools and eyes are different.

Today's Challenge
Student page 77 Have students work in groups of three to share calculators, but ask them to work independently on student page 77. Reassure them that measurements will vary slightly and allow them to switch papers with a partner if they want a second opinion on their measurements.

Answers for student page 77: 1. Ratio 1 should be about 0.64 for every triangle and Ratio 2 should be about 0.84. 2. Lengths of sides will vary, but Ratio 1 should be about 0.5 and Ratio 2 should be about 0.58.

Look Ahead
If students are not seeing patterns emerge, encourage them to find the ratios for some 45°-45°-90° triangles before next time.
Materials
Student page 78

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Work with similar figures. (MOC 376)
Solve proportions. (MOC 434)

Background
To find a missing term in a proportion, use cross products.

Example: The ratio of side $x$ to side $y$ is 0.6; side $x$ is eight inches long. Set up and solve a proportion to find the value of $y$.

\[
\frac{x}{y} = \frac{6}{10}
\]

\[
\frac{8}{y} = \frac{6}{10}
\]

\[
80 = 6y
\]

\[
80 + 6 = y
\]

\[
13.3 = y
\]

Side $y$ is about 13 inches long.

Go Further
Student page 78 Remind students that all measures are approximate, so all computed answers using direct measurements are also approximate.

Answers for student page 78: Check students' work. Possible answers are provided.

Things we discovered:
• Ratio 1 is about the same for every triangle in a set.
• Ratio 2 is about the same for every triangle in a set.
• All the triangles with equal angles are similar.

1. The ratios should be very close to each other.
2. about 17 cm
3. about 23 cm

Assessment
Student self-assessment page 78 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students realize that ratios of corresponding sides of similar figures are equal?
Materials
Student page 79
Geometric solids (optional)

Concepts and Handbook References
Identify pyramids and prisms by their attributes. (MOC 394, 403-404)
Count faces, vertices, and edges of solids. (MOC 393)

Background
A prism is a polyhedron that has two congruent and parallel faces (called bases) joined by (lateral) faces that are parallelograms. Prisms are classified by the shapes of their bases. A pyramid is a polyhedron with at least three triangular (lateral) faces that meet at a point and one other face (the base) that can be any polygon.

Get Started
Draw several prisms and pyramids on the board and discuss the vocabulary associated with each. If you have physical objects to represent the solids, use these as well.

Student page 79 Discuss the words face, vertex, and edge and have students fill in the table on student page 79.

Answers for student page 79:

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>Faces</th>
<th>Vertices</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Prism</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Triangular Pyramid</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Triangular Prism</td>
<td>5</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Hexagonal Prism</td>
<td>8</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Pentagonal Pyramid</td>
<td>6</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

Today’s Challenge
Explain that today the class will be playing a game called “Who Wants to Be the Top Scorer?” Have each student take a blank sheet of paper and draw any prism or pyramid. Have students name the figures they have drawn, then number their papers from 1 through 5.

As you ask each question, have students look at their solids and answer the question. Yes answers will score points.

1. Does your solid have parallel bases? If yes, score 10 points.
2. Does your solid have any right angles? If yes, score 5 points.
3. Does your solid have any triangular faces? If yes, score 9 points.
4. Does your solid have more than ten edges? If yes, score 8 points.
5. Does your solid have exactly four faces? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student draw his or her solid on the board and explain how the points were scored. Discuss why a zero-point score is not possible for this game. (Question 1 gives points for all prisms and question 3 gives points for all pyramids.)

Go Further
Student page 79 Have students solve the riddles and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 79: 2. rectangular prism 3. hexagonal pyramid; base is a regular hexagon 4. Check students’ riddles.

Assessment
Student self-assessment page 79 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students identify pyramids and prisms by their attributes?
Materials
Student page 80

Concepts and Handbook References
Estimate and use logical reasoning to select the correct answer from among four choices. (MOC 528)
Compute with fractions. (MOC 104–107)

Get Started
Tell students that what they know about how addition with mixed numbers works can help them choose the correct answer from among a set of possibilities even without doing all of the computation. Display this test item. Ask students to reason logically to tell you what the correct sum should be.

\[
3\frac{1}{2} + 2\frac{1}{3}
\]

A. 5\frac{1}{6}
B. 6\frac{1}{3}
C. 6\frac{1}{6}
D. 7\frac{1}{2}

Look for reasoning along these lines.

• The mixed numbers in the problem can be rounded up to 4 + 3 and rounded down to 3 + 2, so the sum should be greater than five and less than seven. (D is too great)

• The common denominator of 1/2 and 1/3 must be either sixths or a simplified form of sixths, such as halves or thirds. (A and B have impossible denominators)

• C is correct.

Student page 80 Work with the class on the exercise at the top of student page 155. Matt is correct and the reasoning should involve the denominator of the sum. Hans has an impossible denominator.

Today's Challenge
Student page 80 Have students work independently on student page 80. Tell them that they will be trying to choose the correct sum. Ask them to explain their reasoning for eliminating the incorrect answer choices.

Answers for student page 80: Check students' explanations.
1. C; A is too small and has an impossible denominator, B is too great, D has an impossible denominator
2. B; A and D have impossible denominators, C is too small
3. A; B and C have impossible denominators, D is too small
4. Check students' advice. They should mention bracketing the sum by making a high and a low estimate, then looking at denominators.

Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 80. Encourage them to choose reasonable distracters that can be eliminated based on relative size and denominator. They should write the correct answers on the back. Have them share their problems with a friend, who will reason out (instead of computing) the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 80 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students trust their ability to logically choose the correct sum of mixed numbers from among several distracters?
Materials
Student page 81
Math Maze Cards (Week 17 Activity 81)

Concept and Handbook Reference
Subtract integers. (MOC 136)

Background
You can rewrite any subtraction expression as an addition expression: \( a - b = a + -b \). This is often read, \( a \) minus \( b \) is equal to \( a \) plus the opposite of \( b \). If \( a \) or \( b \) is a negative number, you can still rewrite the subtraction expression as an addition expression.

Get Started
Subtraction of integers is not commutative, meaning \( a - b \neq b - a \). Therefore, when you ask students to find the difference of 4 and \(-2\), you write the expression symbolically as \( 4 - (-2) \). Work through several examples with students.

- Find the difference of 8 and \(-5\).
  \( 8 - (-5) = 8 + 5 = 13 \)
- Find the difference of 2 and 7.
  \( 2 - 7 = 2 + (-7) = -5 \)
- Find the difference of \(-4\) and 6.
  \( -4 - 6 = -4 + (-6) = -10 \)
- Find the difference of \(-9\) and \(-7\).
  \( -9 - (-7) = -9 + 7 = -2 \)

Today's Challenge
Distribute the 18 Math Maze cards for week 17. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have pencil and paper handy for today's Math Maze.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 195.

Student page 81
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 81 in the student book.

Answers for student page 81:
1. 12, -12
2. -4, 4
3. -5, 5
4. -4, 4
5. 7, -7
6. -18, 18
7. -4, 4
8. -19, 19

Go Further
Student page 81
Have students work with a partner to compare the expressions and values in exercises 1–8.

Answers for student page 81:
9. The order in which the subtraction occurs is reversed. 10. The answers are opposites of one another.

Assessment
Student self-assessment page 81
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Do students comfortably use mental math as they find the difference of two integers?
**Materials**
Student page 82
Tape measure or meter stick
Calculators

**Concepts and Handbook References**
Measure in metric units. (MOC 535, 537)
Use ratios. (MOC 424–426)
Make a scatter plot. (MOC 305)
Discuss greatest possible error and accuracy. (MOC 572)

**Background**
*Greatest possible error* is half of the unit of measure used to make a measurement. When you use a ruler to measure, you round your measure to the nearest ruler mark, so if you are measuring in inches, the true measure could be anywhere between $\frac{1}{2}$ inch less and $\frac{1}{2}$ inch more than the stated measure.

A *scatter plot* shows two related sets of data plotted as ordered pairs on a coordinate grid. The axes are labeled to reflect the range of the two sets of data.

**Get Started**
Ask some students how tall they are. Many of them will respond in feet and inches. Very few will probably respond in centimeters. Tell the class that today they are going to work in centimeters. Explain that measurements like height or shoe length are not exact, but are good approximations. Have a student volunteer stand next to a wall. Demonstrate how to find his or her height by placing the palm of your hand on the crown of the head, fingers touching the wall, then measuring from the floor to your fingers. Write your result where students cannot see it. Then, have several other students measure the same person, each writing the result but not revealing it. Compare results. It is likely that the measures will vary slightly. Talk about *greatest possible error* and be sure that students understand that they should expect errors in the range of $\pm 0.5$ units for any direct measurement.

**Today's Challenge**
**Student page 82** Have students work in small groups to share calculators, but work independently to answer the questions and produce a scatter plot. Help struggling students start their scatter plots.

**Answers for student page 82:**
1. $7.3, 7.2, 7.1, 7.0, 6.7, 7.6, 7.6, 7.4, 7.3, 7.5$
2. 0.5 centimeters
3. Check students’ graphs. The height should increase as the shoe length increases.

**Look Ahead**
Suggest that interested students collect and graph their own height/shoe-length data to compare to today's work.
Materials
Student page 83
Calculators (optional)

Concepts and Handbook References
Find the mean for a set of data. (MOC 274)
Graph a line. (MOC 247–250)
Solve simple equations. (MOC 239–241)
Use the Look for a Pattern problem-solving strategy. (MOC 484)

Background
See page 67 for a review of graphing linear equations.

Get Started
Be sure that students have their completed graphs for activity 82. Review how to graph a line given its equation. You may also want to review how to find the mean of a set of data. (sum of numbers ÷ number of numbers)

Go Further
Student page 83 Have students work in pairs. They may not need calculators, but it would be nice if they had them available.

Answers for student page 83: Check students’ work. Possible answers are provided.

Things we discovered:
• The ratio of Height to Shoe Length is between seven and eight for most people in the data set.
• The graph of the data indicates that as the shoe length increases, so does the height.

1. 7.3; check students’ graphs

2. The line $y = mx$ seems to smooth out the scattered points on the plot.  

3. 182.5 cm

Assessment
Student self-assessment page 83 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students graph points on a coordinate plane? Can they solve and graph linear equations of the form $y = ax$?
Materials
Student page 84
Blank paper (heavyweight if possible) or index cards

Concept and Handbook References
Name and sketch common three-dimensional figures. (MOC 400, 405, 410, 415, 420)

Get Started
Review the names of common geometric solids with the students. Draw these figures on the board.

Remind students that prisms and pyramids are named by their bases, so a triangular prism has triangles for bases and a pentagonal pyramid has a pentagon for a base. Share strategies for sketching three-dimensional figures.

Today's Challenge
Student page 84 Have students look at student page 84. Explain that there are two columns for each problem. The first column contains the name of a common geometric solid and the second column is a sketch of the solid. Have students fill in the table.

Answers for student page 84: Check students’ work to make sure the figures are correctly drawn and labeled.
1. 2. pentagonal pyramid
3. triangular prism 4. 
5. hexagonal prism
6. 
7. 
8. rectangular pyramid
9. 
10. rectangular prism

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 20 small pieces of paper or 20 index cards. Have the students use one slip of paper or card to copy the information from each box on student page 84.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a 4 × 5 array. The first player turns over any two cards. If the cards match (show a geometric solid and the correct name), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 84 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students correctly identify and sketch three-dimensional figures?
Materials
Student page 85

Concepts and Handbook References
Use tree diagrams and the counting principle. (MOC 457–460, 464)
Look for efficient alternative methods for solving problems. (MOC 494)

Background
The counting principle states that if an event can occur in a ways and a second event can occur in b ways, then the two events can occur together in ab ways.

Get Started
Pose this problem.

A clothes manufacturer makes a shirt in four different colors (red, green, yellow, and black), and the pants to go with the shirts in two colors (black and tan). If you bought shirts and pants in all of the possible colors, how many different outfits would you have?

Ask students how they might solve this problem. Some will suggest making a list of all possible outcomes. A tree diagram is a good way to do this.

<table>
<thead>
<tr>
<th>Shirts</th>
<th>Pants</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>T</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>T</td>
</tr>
<tr>
<td>Y</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>T</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>T</td>
</tr>
</tbody>
</table>

Tell students that there is another way to find the number of outfits possible without listing them.

Explain that the **counting principle** is a fast way to calculate the number of outcomes. In this case, since there are four ways to select a shirt and two ways to select a pair of pants for every shirt, then there are \(4 \times 2\) or eight ways to select an outfit.

**Student page 85** Have the students work in pairs to solve the problem at the top of student page 85. Discuss how they solved it with the class. (There are three ways to pick a letter; for each of them, there are two ways to flip the coin, so there are \(3 \times 2\) or six possible outcomes to picking a letter then flipping a coin.)

Today's Challenge
**Student page 85** Keep students in pairs. Encourage them to compute to find their answers, but suggest they start tree diagrams if they are not sure of the factors they need. If students struggle with exercise 4, remind them that zero is a multiple of every number.


Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to the ones on student page 85. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
**Student self-assessment page 85** Have students circle one of the two choices to describe how they feel about the activity.

**Assessment tip** Can your students use the counting principle?
Materials
Student page 86
Math Maze Cards (Week 18 Activity 86)

Concepts and Handbook References
Review basic measurement facts. (MOC 535–536)
Find fractional parts of numbers. (MOC 161)

Get Started
Discuss equivalent units of measure. A door’s height could be measured in different units. If the door is 6 feet tall, it is also 72 inches tall since there are 12 inches in one foot and 6 × 12 = 72. Ask about equivalents for these measures.
• \( \frac{1}{2} \) of an hour equals how many minutes? (30)
• \( 2\frac{1}{2} \) quarts equals how many cups? (10)
• \( 3\frac{1}{4} \) dollars equals how many nickels? (65)
• \( 1\frac{1}{2} \) pounds equals how many ounces? (28)

Today’s Challenge
Distribute the 18 Math Maze cards for week 18. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 196.

Student page 86 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 86 in the student book.

Answers for student page 86: 1. 72 2. 42 3. 25 4. 45 5. 18 6. 10 7. 48 8. 18 9. 14 10. 250 11. 6

Go Further
Student page 86 Have students recall divisibility rules in order to complete the table.

Answers for student page 86: 12. yes; yes; no; yes; no; yes; yes 13. The denominators are all factors of 36.

Assessment
Student self-assessment page 86 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students know equivalents for common units of measure?
Materials
Student page 87
Overhead projector with transparent grid (optional)

Concept and Handbook References
Approximate the areas of circles. (MOC 347, 375)

Get Started
Discuss the meaning of the area of a plane figure. (Area is the number of square units contained by the figure.) Then, construct this figure on the board or a transparency.

Work with students to count squares to find the approximate area of the circle. (One way: add one whole square + four almost-squares + four less-than-half squares to get a sum of about seven squares.)

Today's Challenge
Student page 87 Have students work independently on this activity.

Answers for student page 87: Check students’ work. Answers should be near those provided. 1. 27 square units 2. 48 square units 3. 192 square units 4. 75 square units 5. 108 square units 6. 12 square units 7. 147 square units 8. 3 square units 9. Check students’ work.

Look Ahead
Tell students that next time you will compare a scatter plot of their results with a graph of the formula for the area of a circle.
Materials
Student page 88
Calculators

Concepts and Handbook References
Make a scatter plot. (MOC 305)
Graph equations of the form \( y = ax^2 \).
(MOC 251–252)
Use the Look for a Pattern problem-solving strategy
to discover relationships. (MOC 484)
Compare graphs of linear and quadratic equations.
(MOC 246)

Get Started
Make sure that students have their data from activities 83 and 87 available.

Things we discovered:
• The graph of \( y = \pi r^2 \) smoothes out the points in
  the scatter plot.
• The graph of \( y = \pi r^2 \) is not a line, but a curve.

3. about 2.7 centimeters 4. about 12.56 square
  centimeters 5. The only difference is that the equa-
  tion with a squared variable produces a curve while
  the equation with a variable in the first power pro-
  duces a line.

Assessment
Student self-assessment page 88 Have students circle one of the two choices to describe how they feel about this activity.
Assessment tip Can students graph non-linear equations?

Go Further
Student page 88 Ask students to work in small
groups with access to calculators. Have them study
activities 83 and 87 to complete activity 88.

Answers for student page 88: Check students’ work.
Possible answers are provided.

1–2. Check students’ graphs. \( y = \pi r^2 \) is shown.

![Graph showing area in square units vs. radius](image-url)
Materials
Student page 89
Math Jumble activity poster, digit cards/shape cards

Concept and Handbook References
Study characteristics of plane figures.
(MOC 351–352, 358, 363, 389)

Background
Polygons are named by the number of sides they have: triangle (3), quadrilateral (4), pentagon (5), and hexagon (6). In addition, special words are used to suggest additional attributes. For instance, a regular polygon has all sides congruent and all angles congruent. Scalene means no sides are congruent. Isosceles means at least two sides are congruent.

A figure has line symmetry if the part of the figure on one side of a line is the mirror reflection of the part of the figure on the other side of the line.

Get Started
To review the names and properties of common polygons, draw these figures on the board.

1. \( \triangle \)
2. \( \square \)
3. \( \bigcirc \)
4. \( \triangle \)

Discuss the attributes of each figure.
- Figure 1: Right isosceles triangle (one right angle, two congruent angles, two congruent sides, one line of symmetry)
- Figure 2: Rectangle (two pairs of parallel and congruent sides, four right angles, two lines of symmetry)
- Figure 3: Regular hexagon (all sides congruent, all angles congruent, three pairs of parallel sides, six lines of symmetry)
- Figure 4: Pentagon (one line of symmetry, two right angles, three congruent sides)

Today’s Challenge
Use the shape and 0–9 digit cards to construct this poster.

Explain that the object of today’s Math Jumble is find two adjacent figures that share a common attribute associated with a number next to them. The number may be used more than once and a figure may also be used again.

Adjacent figures are associated with a number next to them. For example, Figures I and E in the right column are next to the number one. Each figure has one right angle. Figures E and G in the right column are next to the number three. Each figure has three sides. Attributes may relate to sides, angles, lines of symmetry, or diagonals.

Student page 89 Have students use the Math jumble on student page 89 to find more strings of a digit and two figures.

Answers for student page 89: Check students’ work. Possible answers are provided. The equilateral triangle (A) and pentagon (K) have three congruent sides. The isosceles trapezoid (H) and isosceles triangle (G) have one line of symmetry. The square (B) and rhombus (L) have 4 congruent sides. The kite (M) and rectangle (D) have two pairs of congruent sides.

Go Further
Student page 89 Have students use the grid on the student page to create a Math Jumble to share with a friend.

Answers for student page 89: Grids and answers will vary. Check students’ explanations.

Assessment
Student self-assessment page 89 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students identify common attributes shared by polygons?
Materials
Student page 90
Calculators

Concepts and Handbook References
Make a box-and-whisker plot. (MOC 302)
Find the first, second, and third quartiles and the range from a data set. (MOC 277, 272)
Be careful of copying errors when organizing data. (MOC 532)

Background
A box-and-whisker plot is a visual display of data using the first, second (median), and third quartiles and the least and greatest data values. The first quartile is the median of the lower half of the data. The third quartile is the median of the upper half of the data.

Get Started
Place the following data on the board and tell the class that it represents number of points scored at last year’s games by the boy’s basketball team at a local middle school.

47 59 53 64 42 61 56 52 70 71 61 41 49 38 60 47 52 55 36 53

Have a volunteer write the scores in order, greatest to least. Emphasize that if they miss any scores or copy any incorrectly, they are likely to affect their box-and-whisker plots and answers to any questions about the data. Find the median (53), first quartile (47) and the third quartile (60.5).

71 70 64 61 60 59 56 55 53 53 52 52 49 47 47 42 41 38 36

Discuss how the median is the middle score in a data set arranged in order. In this case the median is between the tenth and eleventh scores, since there are 20 scores. Since those scores are both 53, the median is also 53. If those scores had been different, the median would have been the mean of those two scores. Indicate that the first and third quartiles are the medians for the lower half of the scores and the upper half of the scores, respectively.

Go on to review how to construct a box-and-whisker plot using the three quartiles, the least score, and the greatest score.

Note that there are no numbers on the box-and-whisker diagram, but that a number line below it completes the plot by providing numerical references to parts of the diagram.

Student page 90 Have students work in pairs to organize the data at the top of student page 90. Then, have a pair write the differences in order and tell how they kept track of the data as they organized the list. (45, 35, 32, 27, 23, 17, 15, 14, 13, 10, 7, 7, 4, 3, 1)

Today’s Challenge
Student page 90 Have students stay in pairs or small groups with access to calculators as they complete Today’s Challenge. Be sure that they carefully check their computation when they find the mean. It is easy to mis-key an entry when there are so many addends.

Answers for student page 90: 1. 44 2. 17 3. Check students’ explanations. First quartile: 7; second quartile: 14; third quartile: 27

5. Check students’ advice. They might mention the need to take extra care accounting for all pieces of data.

Go Further
On a separate sheet of paper, have each student write his or her name and create a data set similar to the one on student page 90. They should draw the related box-and-whisker plot on the back. Have them share their data with a friend, who will organize it and draw a plot, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students feel confident finding quartiles for a large data set?
Materials
Student page 91
Math Maze Cards (Week 19 Activity 91)

Concept and Handbook Reference
Review mean, median, mode, and range.
(MOC 272–276)

Background
Three common types of averages, or measures of central tendency, are the mean, median, and mode. The mean is the sum of a set of numbers divided by the number of numbers in the set. It is also called the arithmetic average. The median is the middle number (or the mean of the two middle numbers) of a set of numbers written in numerical order. The mode is the number that occurs most often in a set of data. If all numbers appear the same number of times, there is no mode. There can also be more than one mode. The range is a measure of dispersion, or how spread out the data is. To calculate the range, find the difference between the greatest and least numbers in the data set.

Get Started
Discuss the definitions of mean, median, mode, and range with the students. Then try the following problems.

• What is the mean of 12, 13, and 17?
  \[(12 + 13 + 17 = 42; \frac{42}{3} = 14)\]
• What is the median of 12, 13, and 17? (13)
• What is the mode of 12, 13, and 17? (no mode)
• What is the range of the data 12, 13, and 17?
  \[(17 - 12 = 5)\]
• What are the mean, median, mode, and range for 10, 10, 14, and 18?
  \[(\text{mean: } 10 + 10 + 14 + 18 = 52; \frac{52}{4} = 13; \\text{median: middle two numbers are 10 and 14; } \\text{10 + 14 = 24}; \frac{24}{2} = 12; \text{mode: 10; range: } 18 - 10 = 8)\]

Today’s Challenge
Distribute the 18 Math Maze cards for week 19. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today’s Math Maze.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 197.

Student page 91 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 91 in the student book.

Answers for student page 91: 1. 10 2. 5 3. 15 4. 16 5. 18 6. no mode 7. 25 8. 2 9. 15 10. 9 11. 6.1 12. 17 13. 11.3

Go Further
Student page 91 Have students pay special attention to the 1000 in exercise 15.

Answers for student page 91: 14. 10\(\frac{1}{2}\); 10 15. 340; 10; 990 16. The large number (1000) has a greater impact on the mean than on the median. The median did not change since the middle number remained the same. The mean got much larger since the sum was much larger. 17. Check students’ work. Possible answer: 10, 25, 25

Assessment
Student self-assessment page 91 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students know how to calculate the mean, median, mode, and range for a set of numbers?
Materials
Student page 92
Calculators
Rulers

Concepts and Handbook References
Find area. (MOC 366, 375)
Use the Look for a Pattern and Draw a Diagram
problem-solving strategies. (MOC 484, 483)
Use percent. (MOC 444)
Find probability. (MOC 465, 469)

Background
A circle inside a square, with diameters touching the
square is inscribed in the circle. The length of a side
of the square is also the length of a diameter of the
circle.

Area (square) = \( s^2 \)
Area (inscribed circle) = \( \pi r^2 \) where \( r = \frac{1}{2} s \).

This activity assumes no skill at aiming the dart. Skill
would change the probability of a dart landing in
the circle.

Get Started
Draw a circle inscribed in a square on the board.

Ask students to compute the area of both figures
given only the side length of the square. (square:
144 square inches; circle: 36\( \pi \) or about 113 square
inches) Once students have computed both areas,
have them compute the approximate percent of
the square represented by the area of the circle.
(\( \frac{36\pi}{144} \approx 78\% \))

Today’s Challenge
Student page 92 Pass out calculators to groups of
three students. Students should use \( \pi \approx 3.14 \) to
compute the circle areas. If they need help, you
may wish to use these hints.

- How does the radius of the circle in Target A com-
pare to the length of a side of the square? (\( \frac{1}{2} \))
- How does the radius of a circle in Target B com-
pare to the length of a side of the square? (\( \frac{1}{2} \))
- Consider landing on one square unit of the
circular part of the target as one outcome. Then
the probability of earning a point is \( \frac{\text{square units in circle}}{\text{square units in square}} \).

Answers for student page 92: 1. A. about 452.16
square inches. B. 576 square inches 2. A. about
113.04 square inches B. about 452.16 square
inches C. 576 square inches 3. about 78.5%;
about 78.5% 4. Check students’ work. The proba-
bilities are the same.

Look Ahead
Tell students that next time they will use their
results to compare targets and to extend their
findings to other targets.
Data 2

Analyze Patterns

Week 19 Activity 93

Materials
Student page 93
Coins or compasses
Rulers

Concepts and Handbook References
Find area. (MOC 366, 375)
Use the Look for a Pattern and Draw a Diagram problem-solving strategies. (MOC 484, 483)
Use percent. (MOC 444)
Find probability of independent events. (MOC 465, 469)

Get Started
Ask for volunteers to describe how to find the probability for earning points on each target. Be sure students understand that each dart has to hit the target to be counted.

Go Further
Student page 93 Let students work in pairs. Provide some way for them to draw accurate squares and circles.

Answers for student page 93: 1. about 78.5%
2. It does not matter: the probabilities are the same.
3. The area of an inscribed circle is the same percent (78.5%) of the area of the circumscribed square regardless of the size of the circle. 4. Check students' work. Since circles are adjacent, the sum of the diameters of a row or column of circles always equals the length of a side of the square, so

\begin{align*}
\text{Area (one-circle)} &= \pi(\frac{1}{2} s)^2 = \frac{1}{4} \pi s^2 \\
\text{Area (four-circle)} &= 4\pi(\frac{1}{4} s)^2 = \frac{1}{4} \pi s^2 \\
\text{Area (nine-circle)} &= 9\pi(\frac{1}{6} s)^2 = \frac{1}{4} \pi s^2 \\
\text{Area (sixteen-circle)} &= 16\pi(\frac{1}{8} s)^2 = \frac{1}{4} \pi s^2
\end{align*}

Assessment
Student self-assessment page 93 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students explain why the probability of earning a point is the same for any target, regardless of the number of squares?
Materials
Student page 94
Slips of paper containing shapes
Tracing paper
Rulers

Concept and Handbook Reference
Identify lines of symmetry in common figures.
(MOC 389)

Get Started
Draw these polygons on the board. Do not include the lines of symmetry until you have discussed them with the students.

A circle has an infinite number of lines of symmetry.

Ask students to describe the symmetry of each figure. If students suggest additional figures, draw them on the board as well. Ask students to draw figures with zero, one, and two lines of symmetry. (Possible answers: any scalene figure; heart; non-square rhombus) Ask what figure would have exactly six lines of symmetry. (regular hexagon)

Today’s Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with one of these figures drawn on it and labeled.

You do not need to use all of the shapes, but be sure that one student receives the scalene triangle, since that polygon will be the “Fantastic Finalist.”

Have all students hold their shapes and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
• If your shape has more than three lines of symmetry, sit down. (square, regular hexagon)
• If your shape has exactly one vertical line of symmetry, sit down. (isosceles trapezoid, kite, smiley face, heart)
• If your shape has exactly one horizontal line of symmetry, sit down. (concave kite)
• If your shape has three lines of symmetry, sit down. (equilateral triangle)
• If your shape has two lines of symmetry, sit down. (rectangle, rhombus, oval)

At this point, only the student holding the scalene triangle should still be standing. That student is the “Fantastic Finalist.”

Go Further
Student page 94 Hand out tracing paper and rulers so students can test the figures on student page 94 for lines of symmetry.

Answers for student page 94:
1–5.
6. equilateral triangle
7. Check students’ work. Possible answers: all congruent angles, three congruent sides, three lines of symmetry, each angle measures 60°, sum of interior angles is 180°

Assessment
Student self-assessment page 94 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students identify the lines of symmetry in common figures?
Materials
Student page 95

Concepts and Handbook References
Estimate and use logical reasoning to select the correct answer from among four choices. (MOC 428)
Compute with fractions. (MOC 163)

Get Started
Tell students that what they know about how multiplication with mixed numbers works can help them choose the correct answer from among a set of possibilities even without doing all of the computation.
Display this expression. Ask students to reason logically to tell you what the correct product should be.

\[
4 \frac{1}{2} \times 2 \frac{1}{4}
\]

A. \(10\frac{1}{6}\)
B. \(8\frac{3}{8}\)
C. \(10\frac{1}{2}\)
D. \(1\frac{1}{5}\)

Look for reasoning along these lines.
• The problem can be rounded to \(5 \times 2\), so the answer should be close to 10.
• The partial product from the two fraction parts of the factors, \(\frac{1}{2} \times \frac{1}{4}\), cannot result in any fractions but those with denominators of two, four, or eight, regardless of the numerators.
• Choice B is too small since it barely accounts for the whole-number multiplication.

Student page 95 Work with the class on the exercise at the top of student page 95. Frank is correct and the reasoning should involve the denominator of each answer. Ernest has an impossible denominator.

Today's Challenge
Student page 95 Encourage students to choose the correct product without computing. Ask them to explain their reasoning for eliminating the incorrect answer choices.

Answers for student page 95: Check students’ explanations. Samples are given. 1. D; A and C have impossible denominators, and B is too small 2. B; both C and D have impossible denominators, A is too small 3. B; A and D have impossible denominators, C is too great. 4. Check students’ advice. They might mention checking denominators.

Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 95. Be sure that they think about denominators and relative size of the products as they write their answer choices. They should write the correct answers on the back. Have them share their problems with a friend, who will reason out (instead of computing) the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 95 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students trust their ability to logically choose the correct product for two mixed numbers from among several distracters?
Materials
Student page 96
Math Maze Cards (Week 20 Activity 96)

Concepts and Handbook References
Subtract fractions. (MOC 132–135)
Order fractions. (MOC 041)

Background
See page 46 for a review of the use of common denominators.

Get Started
Talk about the role of the numerator and denominator in fractions. Then, discuss the need for a common denominator when subtracting (or adding) fractions. Pose these problems and ask students to explain their thinking.

• What is the difference of \( \frac{5}{6} \) and \( \frac{3}{4} \)?
  You need to find a common denominator.
  A common denominator is 12, so another way of expressing this problem is:
  \[
  \frac{5}{6} \rightarrow \frac{5 \times 2}{6 \times 2} = \frac{10}{12} \\
  \frac{3}{4} \rightarrow \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \\
  \frac{1}{12} 
  \]

• What is the difference of \( \frac{11}{4} \) and \( \frac{5}{3} \)?
  You need to find a common denominator.
  A common denominator is 12, so another way of expressing this problem is:
  \[
  \frac{11}{4} \rightarrow 1 + \left( \frac{1 \times 3}{4 \times 3} \right) \rightarrow \frac{12}{12} + \frac{3}{12} = \frac{15}{12} \\
  \frac{1}{3} \rightarrow \frac{1 \times 4}{3 \times 4} \rightarrow \frac{4}{12} \\
  \frac{11}{12} 
  \]

Today's Challenge
Distribute the 18 Math Maze cards for week 20.
Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today's Math Maze.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, "Who has the card with the answer to the question just read?" Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 198.

Student page 96
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 96 in the student book.

Answers for student page 96: 1. \( \frac{1}{12} \) 2. \( \frac{1}{3} \) 3. \( \frac{3}{4} \) 4. \( \frac{7}{12} \) 5. \( \frac{7}{12} \) 6. \( \frac{5}{3} \) 7. \( \frac{7}{12} \) 8. \( \frac{1}{6} \)

Go Further
Student page 96
Have students share their own octagon puzzles with a friend.

Answers for student page 96: 9. \( \frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{7}{12}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4} \) 10. Check students’ work.

Assessment
Student self-assessment page 96
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Do students consistently remember to find a common denominator when subtracting fractions?
Materials
Student page 97
Calculators
Chart paper and markers

Concepts and Handbook References
Use the Guess, Check, and Revise problem-solving strategy. (MOC 478)
Estimate to find square roots. (MOC 078)

Background
The standard pencil-and-paper algorithm for finding the square root has been all but abandoned since the invention of low cost calculators. The square root of any number is a keystroke away with a calculator. As with any mathematical process, however, it is beneficial if students can estimate their answers before calculating the answer.

Get Started
Post the number 9409 on the board. Discuss a reasonable estimate of the square root. (Between 90 and 100 because 9409 is between 8100 and 10,000. A good first guess is 95. 95² = 9025, which is too low. Try 98. 98² = 9604, too high. Try 97. 97² = 9409.) Then, work with students to make a master list of benchmark square roots for use in Today’s Challenge.

Today’s Challenge
Student page 97 Have students work in small groups, using calculators for multiplying and dividing, but ask them not to use the square root key. Tell them to keep track of each of their divisors in the guess and check process as they find each square root. Their numbers will be used next time to help streamline their guess, check, and revise method. Students should look to improve their strategy as they solve each problem.

For students having difficulty with finding the square root in a few rounds of guess and check, suggest that they think about these points as they revise for the next round.
• As the divisor of a number increases, the quotient decreases.
• Use benchmark square roots for some of the large numbers.

Answers for student page 97: 1. 85 2. 34 3. 39 4. 67 5. 52

Look Ahead
Prepare a large chart of the table on student page 97 for use next time.

\[
\begin{align*}
\sqrt{900} &= 30 \\
\sqrt{1600} &= 40 \\
\sqrt{2500} &= 50 \\
\sqrt{3600} &= 60 \\
\sqrt{4900} &= 70 \\
\sqrt{6400} &= 80 \\
\sqrt{8100} &= 90
\end{align*}
\]
Materials
Student page 98

Concepts and Handbook References
Use the Guess, Check, and Revise problem-solving strategy. (MOC 478)
Estimate to find square roots. (MOC 078)

Get Started
Have students post their guess and check divisors for the exercises on student page 98. Analyze briefly how the second guess should be a number between the quotient and original divisor. Sir Isaac Newton’s idea was to place the new guess exactly halfway between the divisor and quotient by using the mean of the divisor and quotient as the next guess. Demonstrate Newton’s use of successively closer approximations to find the square root of 9409.

Example:

<table>
<thead>
<tr>
<th>Try 90</th>
<th>9409 ÷ 90 = 105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take the mean of your guess and the rounded quotient.</td>
<td>105 ÷ 90 = 195</td>
</tr>
<tr>
<td>195 ÷ 2 = 98</td>
<td></td>
</tr>
<tr>
<td>Try 98</td>
<td>9409 ÷ 98 = 96</td>
</tr>
<tr>
<td>Take the mean of your guess and the rounded quotient.</td>
<td>98 ÷ 96 = 194</td>
</tr>
<tr>
<td>194 ÷ 2 = 97</td>
<td></td>
</tr>
<tr>
<td>Try 97</td>
<td>9409 ÷ 97 = 97; 97 is the square root of 9409</td>
</tr>
</tbody>
</table>

Go Further
Student page 98 Have students use Newton’s method to complete student page 98.

Answers for student page 98: 1. 56 2. 88 3. 63 4. about 41.2 5. about 68.7 6. Either Yes, or No is acceptable if the explanation is sensible.

Assessment
Student self-assessment page 98 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students analyze their estimates in order to make more efficient estimates on the next attempt?
Materials
Student page 99
Calculators (optional)

Concepts and Handbook References
Compute sales tax and discounts. (MOC 443)
Make change using the fewest coins and bills. (MOC 118, 089)
Compare decimal numbers. (MOC 018)

Background
Discounts and sales taxes are stated as percents.
Write the percent as a decimal and multiply by the price of the item. Discounts are then subtracted from the item price and sales taxes are added to the item price.

Get Started
Write $6.50 on the board. Ask students to determine the final cost of a $6.50-item if tax is 8%. ($6.50 × 0.08 = $0.52; $6.50 + $0.52 = $7.02) For purposes of this activity, consider the change one is likely to find in a cash register, so discount uncommon denominations: two-dollar bills, dollar coins, and half-dollar coins. Ask students to determine the fewest common coins and bills that can be used to make the final cost of the $6.50-item. (a five-dollar bill, two one-dollar bills, two pennies) Next ask students to tell you the amount of change they would receive if they purchased the item with a ten-dollar bill. ($2.98) Ask students to determine the fewest common coins and bills that can be used to make that amount. (two one-dollar bills, three quarters, two dimes, three pennies.) Repeat these steps with another item that is discounted 25%.

Student page 99 Have students use the same steps to answer the questions in the Get Started section of page 99 in their books.

Answers for student page 99: 1. no 2. no 3. yes 4. yes 5. no 6. yes 7. yes 8. no

Today's Challenge
Explain that today you will be playing a game called "Who Wants to Be The Top Scorer?" Have each student take a blank sheet of paper and write any money amount less than $20, then ask them to number their papers from 1 through 5. As you ask each question, have students look at their money amounts and answer the question. Yes answers will score points.
1. Is your amount greater than $17.50? If yes, score 10 points.
2. Is your amount less than $2.50? If yes, score 5 points.
3. Can your amount be made with fewer than five coins and two bills? If yes, score 9 points.
4. Can your amount be made with no nickels? If yes, score 8 points.
5. Can you receive a 25% discount and have less than $1.00 left? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student write his or her money amount on the board and explain how the points were scored. If any student earned no points, have him or her explain the scoring, otherwise work together to create a zero-point amount. (Possible zero-point amount: $14.93)

Go Further
Student page 99 Have students solve the riddle and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 99: 9. 3 one-dollar bills, 1 quarter, 3 pennies. 10. Check students' riddles.

Assessment
Student self-assessment page 99 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students compute a sales tax and discount for an item? Can students make change using the fewest coins and bills?
Materials
Student page 100
Chart or overhead transparency of blank fill-in grid (save for future activities)

Concepts and Handbook References
Use percents. (MOC 443)
Use a machine-scoring style fill-in grid. (MOC 532)

Get Started
Display the blank fill-in grid and demonstrate its use. Be sure students understand that they write their answer in the top row, then fill in the digit, decimal point, or fraction bar that matches what they’ve written at the top of each column. Since the grid does not have a dollar sign, money is written as a decimal number. This means that $57.80 must be written as 57.8 if the answer grid has only four columns.

Write this problem on the board.
Jamie works at a pet shop. She gets a 20% employee discount off the price of dog food. The regular price is $9.49. How much does she save with her discount? ($1.90) How much does she pay? ($7.59)

As part of the discussion, clarify the following concepts.
• A discount is deducted from the original price. You can simplify the computation by subtracting the discount percent from 100%, then multiplying by the original price.
• A tax or a surcharge is added to the price. You can simplify the computation by adding the tax percent to 100%, then multiplying by the sale price.
• The amount paid for an item includes the discount, the tax, or both. If both, compute discount, then tax. DO NOT try to combine all three percents before computing.

Student page 100 Work with students on the problem at the top of student page 100. Before they perform any calculations be sure they explain whether the problem involves a discount or a surcharge. (discount; he paid $57.80)

Today’s Challenge
Student page 100 Ask students to solve the problems on page 100 and enter their answers into the grids at the bottom of the page. Remind them that money is always rounded to hundredths.

Answers for student page 100:
1. 2. 3. 4.

5. Check students’ advice. They might mention being sure that they know whether they are dealing with a discount or a surcharge before starting to compute.

Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 100. They should write the correct answers on the back. Have them share their problems with a friend, who will compute the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 100 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students confidently compute with percents?
Materials
Student page 101
Math Maze Cards (Week 21 Activity 101)
Blocks and pictures of polygons (optional)

Concepts and Handbook References
Recognize and draw plane figures and solids. (MOC 348–370, 394–422)
Recognize symmetry. (MOC 387, 389)

Background
A regular polygon has congruent sides and angles. For example, a square is a regular rectangle and it’s also a regular parallelogram. A rhombus, even though its sides are congruent, is not regular because its angles are not all congruent.

A plane figure has line symmetry if you can fold it such that the two parts match exactly. A plane figure has turn or point symmetry if you can turn it less than 360° such that sides and vertices align perfectly.

Get Started
If possible, display geometric blocks, circular objects, and pictures of polygons. Ask students to identify them, listing important characteristics of each plane figure or solid. Have students look around the room and identify different shapes that they see. They might also give examples of common shapes outside the classroom that everyone would be familiar with such as a stop sign (octagon), beverage can (cylinder), and a child’s kite (kite). Discuss figures that are symmetric and those that are not.

Today’s Challenge
Distribute the 18 Math Maze cards for week 21. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 199.

Student page 101 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity.

Answers for student page 101: Check students’ diagrams. Possible answers are provided.

1. Acute angle
2. Line of symmetry
3. Point symmetry
4. Obtuse angle
5. Line of symmetry
6. Points of symmetry
7. Diameter
8. Cylinder
9. Pyramid
10. Cube

Go Further
Student page 101 Have students sketch their answers in the grid.

Answers for student page 101:

11. A. B. C.
12. A. B. C. D.
13. 5

Assessment
Student self-assessment page 101 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students know the names and attributes of common polygons and solids?
Today's Challenge

Student page 102 Tell students that one of their graphs should begin at zero and the other at a different value. Stress that saving space is usually the reason for beginning a graph at a point other than zero, but that such graphs may be misleading. Allow them to work on a separate sheet of graph paper if they prefer.

Answers for student page 102: Check students’ graphs. Possible graphs are given.

1. A Games Won in a Season
   - Pumas
   - Stallions
   - Elks
   - Bisons
   - Wombats

2. A Heights of Players
   - Jamal
   - Ryan
   - Junior
   - Miguel
   - Andy

Look Ahead
Tell students that next time they will look at how the way a graph is constructed can influence how it is read.
Materials
Student page 103
Overhead projector and transparent grid (optional)

Concepts and Handbook References
Make bar graphs. (MOC 287–289, 292)
Study misleading graphs. (MOC 290)

Get Started
Have students review their work on student page 102 to be sure that they have equal intervals and that the bars accurately reflect the quantities they represent. You might ask a few volunteers to share their graphs by drawing them on the board or overhead transparency.

Go Further
Student page 103 Have students work in pairs to analyze their graphs from student page 102, then create an intentionally misleading graph.

Answers for student page 103: Check students’ work. Possible answers are provided.

Things we discovered:
• On both softball graphs, you can tell that the Bisons won more games.
• On Graph A the seasons for all but the Wombats look quite similar while on Graph B, the Bisons’ season looks much better.
• On both basketball graphs, Jamal is obviously the tallest player.
• On Graph A, the players’ heights look similar, but on Graph B, Jamal’s height stands out and Junior looks really short.

1. Starting a graph’s scale at zero helps assure that data are compared fairly.
2. Check students’ graphs.
3. Scale and intervals

Assessment
Student self-assessment page 103 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students analyze bar graphs to be sure comparisons are fair?
Materials
Student page 104
Blank paper (heavyweight if possible) or index cards

Concept and Handbook Reference
Solve for the missing term in a proportion.
(MOC 434)

Background
A proportion is two equal ratios. In the proportion \( \frac{a}{b} = \frac{c}{d} \) (read \( a \) is to \( b \) as \( c \) is to \( d \)), \( a \) and \( d \) are called the extremes and \( b \) and \( c \) are called the means.

There are several ways to solve for the unknown in a proportion. One method is to use cross products: if \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \) (the product of the extremes equals the product of the means).

Example:
\( \frac{1}{x} = \frac{9}{6} \) (The cross products are \( 4 \times 9 \) and \( 6x \). Set them equal to each other and divide both sides by six to get \( x = 6 \).)

Get Started
Review solving for a missing term in a proportion.

Today’s Challenge
Student page 104 Have students solve each proportion and record their answers in the space provided.

Answers for student page 104: 1. 8 2. 2 3. 6 4. 4 5. 12 6. 9 7. 6 8. 3 9. 7 10. 3 11. 8 12. 7 13. 12 14. 9 15. 2 16. 4

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 16 small pieces of paper or 16 index cards. Have the students use one slip of paper or card to copy the proportion from the left side of each box on student page 104. The students should not copy the answer.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a \( 4 \times 4 \) array. The first player turns over any two cards. If the cards match (show two proportions where \( x \) has the same value), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 104 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students use mental math to solve proportions?
Materials
Student page 105

Concepts and Handbook References
Estimate and compute with fractions.
(MOC 032, 176, 192)
Eliminate incorrect answer choices. (MOC 528–529)

Get Started
Write $5\frac{1}{2} \div \frac{1}{4}$ on the board. Ask students to estimate the quotient. Discuss the approximations students give. Probe for the following concepts.

• If each mixed number is rounded to the next whole number the estimate will be about six ($6 \times 1$).

• Since $5\frac{1}{2}$ is divided by a number between zero and one, the quotient will be greater than $5\frac{1}{2}$.

   $5\frac{1}{2} \div 1 = 5\frac{1}{2}$

   $5\frac{1}{2} \div$ a number greater than one = a number less than $5\frac{1}{2}$

   $5\frac{1}{2} \div$ a number less than one but greater than zero = a number greater than $5\frac{1}{2}$

Have students evaluate the expression. ($7\frac{1}{2}$)

Student page 105 Ask students to look at the first problem on student page 105. Ask volunteers to analyze each answer choice. (A cannot be correct because to divide fractions, you multiply by the reciprocal of the divisor; B and D cannot be correct because the divisor is $\frac{1}{a}$, not $8$; C is a good underestimate.)

Today’s Challenge
Student page 105 Have students work through the exercises on student page 105, ruling out at least two answer choices and giving reasons for doing so. If they must compute to distinguish between the last two choices, suggest that they work backward from the answer choice. (If $a \div b = c$, then $c \times b = a$)


Go Further
On a separate piece of paper have each student write his or her name and create three problems similar to those on student page 105. Have students share their problems with a friend, who is to eliminate some answers by using estimation, and then solve the problems. If there is disagreement, students should edit their problems.

Assessment
Student self-assessment page 105 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students estimate to divide mixed numbers by fractions less than one?
Materials
Student page 106
Math Maze Cards (Week 22 Activity 106)

Concept and Handbook Reference
Find a common percent of a number using mental math. (MOC 442–443)

Background
Students should be able to find the following common percents of a number using mental math: 5%, 10%, 25%, 33 1/3%, and 50%. For example, 10% of a number is 1/10 of the number so the decimal point will move one position to the left. (10% of 180 = 1/10 of 180 = 18). Since 5% is half of 10%, 5% of 180 is half of 10% of 180, or 9.

Get Started
Review finding a percent of a number using mental math.
• How could you figure out 25% of 120? (25% = 1/4, and 1/4 of 120 is 30)
• How could you figure out 50% of 120? (50% = 1/2, and 1/2 of 120 is 60 or 50% is twice 25%, 30 × 2 = 60)
• How could you figure out 33 1/3% of 120? (33 1/3% = 1/3, and 1/3 of 120 is 40)
• How could you figure out 5% of 120? (5% is half of 10%, so find 1/10 of 120 which is 12, then take half of 12 which is 6 or take one tenth of 50%)

Today's Challenge
Distribute the 18 Math Maze cards for week 22. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 200.

Student page 106 When the group has finished playing the game, have students open their books and complete the Today's Challenge activity on page 106 in the student book.

Answers for student page 106: 1. 12; 30; 40; 60 2. 2.4; 6; 8; 12; 3. 4.8; 12; 16; 24 4. 6; 15; 20; 30 5. 3.6; 9; 12; 18

Go Further
Student page 106 Encourage students to use mental math on exercises 6–9.

Answers for student page 106: Check students' strategies. Possible answers are provided. 6. 6; 1.2; 2.4; 3. 1.8; find half of 10% 7. 18; 3.6; 7.2; 9. 5.4; add 10% and 5% 8. 80; 16; 32; 40; 24; double 33 1/3% 9. 90; 18; 36; 45; 27; add 50% and 25%

Assessment
Student self-assessment page 106 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students confident using mental math to find common percents?
Materials
Student page 107
Calculators
Chart paper and markers

Concept and Handbook Reference
Find mean and median. (MOC 273–275)

Get Started
Post the following set of data on the board and ask students to compute the mean and median: 36, 45, 56, 34, 76, 56, 68. Ask for a student volunteer to explain how to find both the mean (53) and the median (56).

Today's Challenge
Student page 107 Have students work in groups of three to share a calculator. Remind them to round to the nearest tenth.

Answers for student page 107: 1. A. 53.4 B. 54 C. Mean goes up to 64, median does not change D. Mean drops to about 48.1, median changes to 51.
2. A. 63.3 B. 65 C. Mean goes up to 77, median does not change D. Mean drops to about 56.4, median drops to 59

Look Ahead
Prepare this chart for use next time.

<table>
<thead>
<tr>
<th>Given Data Set</th>
<th>Mean</th>
<th>Median</th>
<th>Double Week Five Data Mean</th>
<th>Median</th>
<th>Halve Week Five Data Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>36, 54, 51, 59, 74, 42, 58</td>
<td>53.4</td>
<td>54</td>
<td>64</td>
<td>54</td>
<td>48.1</td>
<td>51</td>
</tr>
<tr>
<td>66, 83, 65, 59, 96, 29, 45</td>
<td>63.3</td>
<td>65</td>
<td>77</td>
<td>65</td>
<td>56.4</td>
<td>59</td>
</tr>
</tbody>
</table>
Data 2  Analyze Patterns

Week 22•Activity 108

Materials
Student page 108
Calculators

Concepts and Handbook References
Find mean and median. (MOC 273–275)
Use the Look for a Pattern problem-solving strategy. (MOC 484)

Get Started
Post your chart showing correct answers for page 107. Ask students to discuss with each other how the mean and median of each set changed as the data set changed.

Go Further
Student page 108 Ask students to work in small groups to share calculators as they compute the mean and median for the new data set on student page 108. Have them study their results for pages 107 and 108 to complete this activity.

Answers for student page 108: 1. $0.84
2. $0.82
3. A. $0.71
4. A. $1.08
B. $0.79
5. The mean is more sensitive to extreme values or changes in values in a data set. 6. One way would be to change the number of items in the data set (1, 3, 5: median = 3; 2, 3, 9: median = 3 or 2, 4, 6, 10: median = 5; 2, 3, 7, 10: median = 5) 7. If what is added to one data value that is larger than the mean is also subtracted from a data value less than the mean, then the mean is unaffected as long as the number of pieces of data does not change. For example, given 60 and 20, if 60 is changed to 70 and 20 is changed to 10, then their mean remains unchanged.

Assessment
Student self-assessment page 108 Have students circle one of the two choices to describe how they feel about this activity.
Assessment tips Can students accurately compute mean and median? Are they able to identify the mean as the measure of central tendency that is sensitive to extreme values in a data set?
Materials
Student page 109
Math Jumble activity poster and digit cards

Concept and Handbook References
Compute with fractions. (MOC 104, 132, 160, 187)

Background
Fractions can only be added or subtracted if they have a common denominator.
Examples:
- \[ \frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12} \]
- \[ \frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15} \]

To multiply fractions, multiply the numerators and multiply the denominators.
Example:
- \[ \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6} \]

To divide fractions, multiply the dividend by the reciprocal of the divisor.
Example:
- \[ \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3} \]

Get Started
In today's work students will perform all four operations on fractions. Review the operations as needed. Discuss strategies for making sure that a sum of two fractions will be less than one. For instance, if the first fraction is greater than \(\frac{3}{4}\), the second fraction must be at least the same amount less than \(\frac{1}{2}\). If the first fraction is close to one, the second fraction needs to be closer to zero.

Today's Challenge
Use the 0–9 digit cards to construct this poster.

Explain that the object of today's Math Jumble is to find strings of four digits that can be used as the numerators and denominators of two fractions that can be added or subtracted. The sums or differences must be between zero and one. Students must simplify all answers.

The two fractions can be made with any four adjoining digits (horizontally, vertically, and/or diagonally) on the poster. Digits on the poster can be used more than once, but not in the same expression. For example, the digits in the first column can be used to write \(\frac{3}{7} + \frac{1}{5}\). Students then calculate the sum, \(\frac{24}{45}\), and write it in simplest form, \(\frac{8}{15}\). Record the expressions and their values.

Possible equations:
- \(\frac{2}{3} - \frac{1}{8} = \frac{11}{40}; \frac{3}{9} + \frac{1}{2} = \frac{15}{18} = \frac{5}{6}\)
- \(\frac{3}{4} - \frac{3}{8} = \frac{11}{18}\)

Student page 109 Have students use the Math Jumble on student page 109 to find more strings of digits to make into fractions and use in sums or differences.

Answers for student page 109: Check students' work; possible answers are provided. \(\frac{1}{3} + \frac{1}{5} = \frac{8}{15}\); \(\frac{2}{3} + \frac{3}{4} = \frac{29}{20}; \frac{2}{3} - \frac{1}{2} = \frac{11}{18}\)

Go Further
Student page 109 Have students use the fractions from Today's Challenge in multiplication and division expressions.

Answers for student page 109: Check students' work; possible answers are provided. \(\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}\); \(\frac{2}{3} \times \frac{1}{6} = \frac{4}{30} = \frac{2}{15}; \frac{3}{5} + \frac{3}{5} = \frac{6}{5} = \frac{3}{5} \times \frac{1}{8} = \frac{16}{5} = 3\frac{1}{5}\)

Assessment
Student self-assessment page 109 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students estimate sums and differences of simple fractions? Can students correctly perform all four operations with simple fractions?
Materials
Student page 110
Calculators

Concepts and Handbook References
Work with percents. (MOC 442–443)
Use the Write and Solve an Equation problem-solving strategy. (MOC 482)
Solve multi-step problems. (MOC 488)

Get Started
Sometimes students are put off by complex-looking word problems. Sometimes they compute without reading carefully. Both of these responses are dangerous in a test situation.

Read this problem aloud. Ask them to rewrite each fact as an equation.

Jacob ate lunch at a hamburger restaurant. The tax on fast-food where Jacob lives is 8%. The total food bill, tax included, was $4.86. What was the price of the food?

• tax = 8% \times \text{price}
• \text{price} + \text{tax} = \text{bill}
• \text{bill} = \$4.86

Now work together to write and solve an equation that accurately represents the problem.

\begin{align*}
\text{price} + \text{tax} &= \text{bill} \\
\text{price} + 0.08\text{price} &= 4.86 \\
1.08\text{price} &= 4.86 \\
\text{price} &= 4.5
\end{align*}

Always go back to the original question as you write your answer: The price of the food was $4.50.

Student page 110 Work with the class on the exercise at the top of student page 110. Help students to define each element of the problem. Then combine elements to write an equation.

Possible solution:
shipping and handling = 15\% \times \text{price} \\
\text{price} + 15\% \times \text{price} = \text{total cost} \\
\text{total cost} = 88.48

First, find price.
\begin{align*}
p + 0.15p &= 88.48 \\
1.15p &= 88.48 \\
p &= 76.94
\end{align*}

Next, find 15\% of price.
0.15 \times 76.94 = 11.54

Refer to the question to write the answer
The shipping and handling charge was $11.54.

Today’s Challenge
Student page 110 Tell students to carefully define each part of each problem before writing and solving an equation. Have them sit in small groups with access to calculators.

Answers for student page 110: 1. The tip was $10.96
2. The price was $16.98
3. The late fee was $2.25
4. The price of the shirt was $25.49
5. Check students’ advice. They might comment on breaking up the problem into small steps.

Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 110. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problem, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 110 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students solve multi-step percent problems with confidence?
Materials
Student page 111
Math Maze Cards (Week 23 Activity 111)

Concept and Handbook References
Use mental math and estimation to compute with
decimal numbers. (MOC 088, 116, 143, 173)

Background
See page 11 for a review of reading decimal num-
bers. It is important for students to read decimals
properly as this makes mental math much simpler.

Place value is an important part of operating with
decimals. When you add or subtract decimals, the
same place values are added or subtracted. To
accomplish this with paper and pencil, you line up
the decimal points. When doing mental math com-
putations, students must think about the location
of the decimal point so that like place values are
added or subtracted. To multiply and divide mental-
ly, students will need to compute as if they had
whole numbers and then apply the decimal opera-
tion rules or estimation to locate the decimal point.

Get Started
Begin by asking students the rules for decimal op-
erations. Probe for these points.
• To add (subtract) decimals, line up the decimal
points and add (subtract) as you would with
whole numbers.
• To multiply decimals, multiply as you would with
whole numbers, then make sure the product has
the same number of decimal places as in the sum
of the number of decimals places in the factors.
• To divide decimals, divide as you would with
whole numbers, then count the number of places
in the dividend: that is the sum of the number of
places in the factors. (the quotient and divisor)
• You can also place the decimal point by estimat-
ing the product or quotient.

Practice with the following problems, encouraging
students to use mental math strategies.
• What is the sum of 4.52 and 1.37? (5.89)
• What is the difference of 3.6 and 1.24? (2.36)
• What is the product of 3.5 and 0.4?
  (35 × 4 = 140; three times a number less than
  half will be less than 1.5, so the product is 1.4)
• What is the quotient of 1.25 and 0.5?
  (125 ÷ 5 = 25; 1.25 (the dividend) has two deci-
  mal places, 0.5 (one factor) has one decimal
  place, so the quotient (second factor) also has one
decimal place. 1.25 ÷ 0.5 = 2.5)

Today's Challenge
Distribute the 18 Math Maze cards for week 23.
Each student should receive at least one card, but
since all cards need to be distributed, some students
may need to get more than one card. Use the cards
to play the Math Maze game. Students may wish to
have pencil and paper available for today's Math
Maze.

Instructions for playing Math Maze Ask students to
look at their cards. Ask one student to read the
question that is written on his or her card. Next ask,
"Who has the card with the answer to the question
just read?" Ask that student to read the answer, and
then read the question on his or her card. Play con-
tinues until all questions have been answered. The
last answer to be read should be the answer on the
first student's card.

The correct sequence of questions and answers is
shown on page 200.

Student page 111 When the group has finished
playing the game, have students open their books
and complete the Today's Challenge activity on
page 111 in the student book.

Answers for student page 111: 1. g 2. d 3. f 4. a
5. h 6. b 7. j 8. e 9. c 10. i

Go Further
Student page 111 Have students use patterns to
complete the problems.

Answers for student page 111: 11. 1.32, 13.2, 132,
1320 12. 552, 55.2, 5.52, 0.552

Assessment
Student self-assessment page 111 Have students
circle one of the two choices to describe how they
feel about this activity.

Assessment tip Are students able to apply decimal
operation rules to their mental math skills with
whole numbers?
Materials
Student page 112
Overhead projector and transparent geobord or
4 x 4 grid (optional)

Concepts and Handbook References
Practice spatial visualization with right triangles.
(MOC 358–361)
Use the Pythagorean Theorem. (MOC 359)
Compute with powers and roots. (MOC 076)

Background
The Pythagorean Theorem states that, for any right
triangle, the sum of the squares of the legs equals
the square of the hypotenuse.

\[ a^2 + b^2 = c^2 \]

Get Started
On the board or overhead projector, show students
this segment on a geoboard. Ask them how to find
its length.

Ask a student to volunteer to explain how to com-
pute the length of the segment using the
Pythagorean Theorem. In this case the segment is
the hypotenuse of the right triangle shown here.

Use the Pythagorean Theorem to find the length of
the hypotenuse.

\[ a^2 + b^2 = c^2 \]
\[ 2^2 + 1^2 = c^2 \]
\[ 4 + 1 = c^2 \]
\[ 5 = c^2 \]
\[ \sqrt{5} = c \]

There is no need for students to evaluate \( \sqrt{5} \) with a
decimal approximation.
The task is not to find all of the possible segments
but to find all possible segment lengths.

Today's Challenge
Student page 112 Remind students to use the
Pythagorean Theorem to help them in their search
for all the segment lengths on the geoboard. For
students who are having difficulty with the task,
give the following hints.

• Do not forget to try horizontal, vertical, and
  oblique segments.

• Try different combinations of legs for right trian-
gles and then compute the length of the
  hypotenuse. For example, the hypotenuse of a
  right triangle with legs two and three units long is
  \( \sqrt{13} \) units long.

• Be sure to count distances between pegs, not the
  pegs themselves. The longest horizontal segment
  is four units long.

Answers for student page 112: 1. There are 14 dif-
ferent segment lengths possible. Students may not
find them all until next time.

Look Ahead
Tell students that next time they will compile a list
of all of the segment lengths they found, and then
study the list to decide whether any more segment
lengths are possible.
Materials
Student page 113

Concepts and Handbook References
Practice spatial visualization with right triangles. (MOC 358–361)
Use the Pythagorean Theorem. (MOC 359)
Compute with powers and roots. (MOC 076)
Use the Make a List problem-solving strategy. (MOC 480)

Get Started
Have students post some of the segment lengths they found in activity 112. Have them draw a diagram to show each segment that is the hypotenuse of a right triangle.

Go Further
Student page 113 Have students work in pairs on this activity.

Answers for student page 113:
1. The lengths of vertical or horizontal line segments are one, two, three, and four units. There are ten segments that are hypotenuses of right triangles. They are listed in the table here.

<table>
<thead>
<tr>
<th>Segment Length (in units)</th>
<th>Right Triangle Legs (in units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{2})</td>
<td>1 \times 1</td>
</tr>
<tr>
<td>(\sqrt{5})</td>
<td>1 \times 2</td>
</tr>
<tr>
<td>(\sqrt{10})</td>
<td>1 \times 3</td>
</tr>
<tr>
<td>(\sqrt{17})</td>
<td>1 \times 4</td>
</tr>
<tr>
<td>(\sqrt{8})</td>
<td>2 \times 2</td>
</tr>
<tr>
<td>(\sqrt{13})</td>
<td>2 \times 3</td>
</tr>
<tr>
<td>(\sqrt{20})</td>
<td>2 \times 4</td>
</tr>
<tr>
<td>(\sqrt{18})</td>
<td>3 \times 3</td>
</tr>
<tr>
<td>(\sqrt{25})</td>
<td>3 \times 4</td>
</tr>
<tr>
<td>(\sqrt{32})</td>
<td>4 \times 4</td>
</tr>
</tbody>
</table>

2. Check students’ work. 3. 1, 2, \(\sqrt{2}\), \(\sqrt{5}\), \(\sqrt{8}\)
4. 1, 2, 3, 4, 5, \(\sqrt{2}\), \(\sqrt{5}\), \(\sqrt{8}\), \(\sqrt{10}\), \(\sqrt{13}\), \(\sqrt{17}\), \(\sqrt{18}\), \(\sqrt{20}\), \(\sqrt{25}\), \(\sqrt{26}\), \(\sqrt{29}\), \(\sqrt{32}\), \(\sqrt{34}\), \(\sqrt{41}\), \(\sqrt{50}\)

Assessment
Student self-assessment page 113 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle? Are they able to organize a systematic search?
Materials
Student page 114
Slips of grid paper showing points on a coordinate plane.

Concepts and Handbook References
Locate points on a coordinate plane. (MOC 318–320)
Determine whether coordinate pairs are solutions to an equation. (MOC 242)

Get Started
Review locating points on a coordinate plane.
- The x-axis is horizontal and the y-axis is vertical.
- A point is located using two numbers: distance along the x-axis, then distance along the y-axis.
- An ordered pair gives the x-coordinate and the y-coordinate.
- The origin, where the axes intersect, has coordinates (0, 0).
- Numbers to the left of the origin have a negative x-coordinate.

Example: (6, 4)
names the point six units to the right of zero on the x-axis and four units above zero on the y-axis.
(−4, 2) names the point four units to the left of zero on the x-axis and two units above zero on the y-axis.

Today's Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of grid paper or a copy of the empty coordinate plane on student page 114 with one of these ordered pairs marked but not labeled: (0, 0), (3, 0), (−1, 1), (1, 1), (1, 2), (−5, 3), (−3, 3), (0, 3), (3, 3), (6, 3), (−1, 4), (−3, 6), (0, 6), (6, 6), (−1, 7), (3, 7), (−5, 8), (6, 8).

You do not need to use all of the ordered pairs, but be sure that one student receives the ordered pair (6, 8), since the student holding that ordered pair will be the “Fantastic Finalist.”

Have all students hold their points and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
- If your ordered pair has equal coordinates, sit down. (0, 0), (1, 1), (3, 3), (6, 6)
- If your ordered pair has opposites coordinates, sit down. (−1, 1), (−3, 3)
- If your ordered pair is on the y-axis, sit down. (0, 3), (0, 6)
- If your ordered pair works in the equation \( y + x = 3 \), sit down. (−5, 8), (−3, 6), (−1, 4), (1, 2), (3, 0)
- If \( xy < 40 \), sit down. (−5, 3), (−1, 7), (3, 7), (6, 3)

At this point, only the student holding the point (6, 8) should still be standing. That student is the “Fantastic Finalist.”

Go Further
Student page 114 Have students complete the activity on the student page.

Answers for student page 114: 1–5.

6. (6, 8) 7–8.
8. a rectangle

Assessment
Student self-assessment page 114 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students locate points on a coordinate plane without confusing x- and y-coordinates?
Materials
Student page 115
Calculators

Concepts and Handbook References
Use the Draw a Diagram problem-solving strategy. (MOC 483)
Use the Pythagorean Theorem. (MOC 359)

Background
Refer to page 112 for a review of the Pythagorean Theorem.

Get Started
Tell students that sometimes a problem requires a diagram, but does not provide one. In that case it is important to make a useful diagram before trying to compute the answer. Encourage them to use scrap paper, margins, or the back of a test paper to sketch diagrams for test problems.

Student page 115 Ask students to look at the first problem on student page 115. Ask a volunteer to read the problem aloud. Next, ask students to make a diagram of the problem and ask for a volunteer to post the drawing on the board. Once you have an accurate drawing, discuss how the Pythagorean Theorem will help students to solve the problem. As part of the discussion stress these points.

- Students should be sure that they solve for the proper length, and not simply square and sum two values, assuming they are the lengths of the legs.
- When the result of computation is not a perfect square, it may be left in square root form or written as an approximate value. Students should check directions on a test when it is not multiple choice.

Be sure everyone understands why C is correct.

Solution:
\[ 9^2 + 12^2 = AC^2 \]
\[ 81 + 144 = AC^2 \]
\[ 225 = AC^2 \]
\[ 15 = AC \]

Today’s Challenge
Student page 115 Have students sit in small groups with access to calculators but encourage them to work independently. Have them work through the remaining problems on student page 115. Check to see that students are making correct diagrams and solving for the missing leg or hypotenuse.

Answers for student page 115:

1. C

2. A

3. B

4. D

5. Check students’ advice. They might mention drawing and using careful diagrams.

Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 115. They should write the correct answers on the back. Have them share their problems with a friend, who will draw a diagram, solve the problem, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 115 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students correctly use the Pythagorean Theorem to solve for a missing side length in a right triangle?
Materials
Student page 116
Math Maze Cards (Week 24 Activity 116)

Concepts and Handbook References
Multiply fractions. (MOC 162)
Order fractions. (MOC 041)

Background
To multiply fractions, multiply the numerators and multiply the denominators.
\[
\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}
\]

This diagram represents \(\frac{1}{2} \times \frac{1}{4}\)

This diagram represents \(\frac{1}{4}\)

This diagram represents \(\frac{1}{2} \times \frac{1}{4}\) (\(\frac{1}{2}\) of \(\frac{1}{2}\))

Get Started
Review multiplication of fractions. Discuss the area model for multiplication. To find \(\frac{1}{2} \times \frac{2}{3}\), shade a horizontal half of the rectangle one way and a vertical \(\frac{2}{3}\) of the rectangle a different way. The product is where the two shadings overlap. The name for this region is \(\frac{1}{2}\) which simplifies to \(\frac{1}{3}\).

\[
\begin{align*}
\frac{1}{2} \times \frac{2}{3} &= \frac{2}{6} \text{ or } \frac{1}{3} \\
\frac{1}{3} &= \frac{2}{6} \text{ or } \frac{1}{3}
\end{align*}
\]

Students may also recall that common factors can be divided out before the fractions are multiplied.

Examples:
- \(\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} \times \frac{1}{3}
- \frac{2}{4} \times \frac{3}{5} = \frac{6}{20}
- \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}
- \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}
- \frac{7}{10} \times \frac{1}{2} = \frac{7}{20}
- \frac{7}{10} = \frac{7}{10}

Ask students to find the following products.
- \(\frac{2}{3} \times \frac{1}{4} = ? (\frac{1}{6})
- \(\frac{3}{5} \times \frac{2}{3} = ? (\frac{6}{15})
- \(\frac{7}{10} \times \frac{3}{2} = ? (\frac{21}{20})

Today’s Challenge
Distribute the 18 Math Maze cards for week 24. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for this Math Maze.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 201.

Student page 116
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity.

Answers for student page 116:
1. \(\frac{1}{6}\)  2. \(\frac{1}{5}\)  3. \(\frac{5}{9}\)  4. \(\frac{1}{49}\)  5. \(\frac{1}{9}\)

Go Further
Student page 116
Help students to recognize the least common denominator for all of the fractions, and then either rewrite each or compare to benchmarks 0, \(\frac{1}{2}\), and 1 in order to arrange them from least to greatest.

Answers for student page 116:
4. \(\frac{72}{186}\)  5. \(\frac{1}{6}\)  6. \(\frac{1}{9}\)  7. \(\frac{5}{9}\)  8. \(\frac{1}{4}\)  9. \(\frac{1}{5}\)  10. \(\frac{1}{2}\)  11. \(\frac{5}{25}\)  12. \(\frac{1}{18}\)

Assessment
Student self-assessment page 116
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Do students remember to look for a common factor when multiplying fractions so that the product is in simplest form?
Materials
Student page 117
Overhead projector and transparent geoboard or
3 x 3 grid (optional)

Concepts and Handbook References
Practice spatial visualization of quadrilaterals. (MOC 364)
Use the Make an Organized List problem-solving
strategy. (MOC 480)

Get Started
Draw this square on the board or overhead projector. Confirm that the figure is drawn on a square grid.

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\]

Ask students to identify the figure. Most will readily identify the figure as a diamond, not a square. Ask students for another category of polygon that will accommodate the figure. Quadrilateral and parallelogram are likely answers. Convince them that, since all of the sides are the same length (the hypotenuse of a right triangle with one-unit legs), it is a rhombus. Then, recall that, since each triangle is an isosceles right triangle, its acute angles measure 45°. Your figure has angles that are twice 45°, so it must be a square.

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\]

Today's Challenge
Student page 117 Encourage students to find all possible squares that may be represented on a standard geoboard. For students who find only the squares in traditional (horizontal/vertical) position, give these hints.
- Recall the opening activity with the rotated square.
- Recall activities 112–113, where segments were vertical, horizontal, and oblique.

Answers for student page 117: 1. Possible square sizes are 1 x 1, 2 x 2, 3 x 3, 4 x 4, 2 x 2, \sqrt{5} x \sqrt{5}, 2 x 2, \sqrt{5} x \sqrt{5}, 2 x 2, \sqrt{5} x \sqrt{5}. Do not require that students have all of these until next time.

Look Ahead
If students are not sure whether they have identified all possible square sizes, encourage them to look for more before next time.
Materials
Student page 118

Concepts and Handbook References
Practice spatial visualization of quadrilaterals. (MOC 364)
Multiply square roots. (MOC 076)
Use the Make a List problem-solving strategy. (MOC 480)
Compare and order whole numbers. (MOC 007-009)

Background
\( \sqrt{a} \times \sqrt{a} = a \) when \( a \geq 0 \)

Get Started
Ask for student volunteers to post a few of the squares they found for activity 117. Relate the squares to their side lengths. Standard position squares have side lengths of one, two, three, and four units. Additional squares may be found by considering all possible segment lengths for sides. Refer to activities 112–113, where students found all possible segments for a five-peg x five-peg geoboard.

Go Further
Student page 118 Have students work in pairs. If necessary, tell them that there are more than six possible square sizes.

Answers for student page 118: 1. Position on the grid will vary.

2. Dimensions (in units): \(1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, \sqrt{2} \times \sqrt{2}, \sqrt{5} \times \sqrt{5}, \sqrt{8} \times \sqrt{8}, \sqrt{10} \times \sqrt{10}\) 3. Check students' work. 4. Areas (in square units): 1, 2, 4, 5, 8, 9, 10, 16

Assessment
Student self-assessment page 118 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students persevere in a search when the outcome is uncertain?
Materials
Student page 119

Concept and Handbook Reference
Compare positive and negative numbers.
(MOC 046–049)

Get Started
On the board, draw a giant number line like the one shown on page 119. Together with the students, identify each fractional part of the number line and record the rational numbers, in simplest form, for each mark. You may label all of the integers, followed by the halves, and finally the fourths.

Student page 119 Have students record each rational number on the number line in the Get Started section of student page 119.

Answers for student page 119: 1.

As you ask each question, have students look at their numbers and answer the question. Yes answers will score points.

1. Is your rational number negative? If yes, score 10 points.
2. Is your rational number greater than $-1$? If yes, score 5 points.
3. Is your rational number less than $-1\frac{1}{2}$? If yes, score 9 points.
4. Does the fraction portion of your rational number have a numerator of one? If yes, score 8 points.
5. Is your rational number between $-1\frac{1}{4}$ and $-1$? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student write his or her rational number on the board and explain how the points were scored. Discuss why no zero-point scores are possible for this game. (Everyone will score points from either question 1 or question 2 or both.)

Today’s Challenge
Explain that today the class will be playing a game called “Who Wants to Be the Top Scorer?” Have students use the number line on page 119 and select a rational number from $-2$ through 1. Have each student write his or her rational number on a blank sheet of paper, then ask them to number their papers from 1 through 5.

Go Further
Student page 119 Have students solve the riddles, write and solve their own riddle, and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 119: 2. $-\frac{3}{4}$ 3. $-1\frac{5}{8}$ 4. $-1\frac{7}{8}$ 5–6. Check students’ riddles.

Assessment
Student self-assessment page 119 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students correctly name simplified rational numbers with denominators two, four, and eight?
Materials
Student page 120
Calculators

Concepts and Handbook References
Find percent increase and decrease. (MOC 446–449)
Write a clear, careful constructed response for a multi-step problem. (MOC 531)
Use the Solve a Simpler Problem problem-solving strategy. (MOC 481)

Get Started
Review how to write a good constructed response. (Break down the problem into a series of steps; explain each step, showing computation and diagrams; use the wording of the original problem to help you write your answer.)

Discuss finding the percent of increase and the percent of decrease. To find percent change, compare the difference between the new and original amounts to the original amount.

Examples:
• New price = a
  Original price = b
  Price change = a – b (a > b)
  Percent increase \( \frac{x}{100} = \frac{|a - b|}{b} \)
• New price = a
  Original price = b
  Price change = a – b (a < b)
  Percent decrease \( \frac{x}{100} = \frac{|a - b|}{b} \)

Student page 120
Ask students to work in pairs to study the Get Started problem and its solution. After a few minutes, discuss why this is a clear, complete constructed response. Ask students to suggest alternative ways to respond to the same question. Be sure they understand why a 25% increase followed by a 25% decrease does NOT result in the original amount. (You are taking 25% of a larger number when you compute the decrease.)

Answers for student page 120: 1. Possible answer:
Find the new cost after the first increase.
\[
\text{new cost} = \text{old cost} + 10\% \text{ of old cost} \\
= 100\% + 10\% \text{ of old cost} \\
= 1.1 \times 30.99 \\
= 34.089
\]
Round to reflect the fact that this is money.
\[34.089 \approx 34.09\]

Find the new cost after the second increase.
\[
\text{new cost} = \text{old cost} + 5\% \text{ of old cost} \\
= 100\% + 5\% \text{ of old cost} \\
= 1.05 \times 34.09 \\
= 35.7945
\]
Round to reflect the fact that this is money.
\[35.7945 \approx 35.79\]

Compare the newest cost to the original cost.
\[
35.79 - 30.99 = 4.8
\]

Find the percent increase.
\[
\frac{4.8}{30.99} = 0.154 \times 100 = 15.4\%
\]

Write a clear, complete answer to the original question.
The cost of cable service increased about 15.5% over two years.

2. Check students’ work. Advice should include a caution about percent change followed by another percent change.

Go Further
On a separate sheet of paper, have each student write his or her name and create one problem similar to those on student page 120. They should write the main points of a constructed response and the correct answer on the back. Have them share their problem with a friend, who will write a constructed response, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 120
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Do students understand the care required when computing percent change followed by another percent change?

Today’s Challenge
Student page 120
Have students work in groups of two or three, sharing a calculator. Students can consult with each other, but each student should complete student page 120 independently.
Materials
Student page 121
Math Maze Cards (Week 25 Activity 121)

Concept and Handbook Reference
Multiply fractions and whole numbers. (MOC 161)

Background
You can use an area model to find a fraction of a number. To illustrate $\frac{3}{8}$ of 560, use a rectangle with the dimensions $70 \times 8$, each row of 70 is $\frac{1}{8}$ of 560, so $\frac{3}{8}$ would be three of those rows of 70. Without the model, divide 560 by 8 to get one eighth of 560, then multiply by three since you are looking for three of the eighths. (210)

Get Started
Review finding a fraction of a number.

• How could you figure out $\frac{4}{3}$ of 350? ($\frac{1}{3}$ is 70, so $\frac{4}{3}$ is $4 \times 70$, or 280)

• How could you figure out $\frac{5}{6}$ of 720? ($\frac{1}{6}$ is 120, so $\frac{5}{6}$ is $5 \times 120$, or 600)

• What about $\frac{2}{3}$ of 120? (It is easier if you think of $\frac{2}{3}$ as $\frac{1}{3}$ because 12 is evenly divisible by 4, but not by 8; $\frac{1}{4}$ of 120 is 30.)

Next, complete these puzzles with the class (students will need pencil and paper).

• What is the first $\frac{5}{6}$ of maybe added to the last $\frac{5}{6}$ of fifth? (math)

• Take the first $\frac{1}{3}$ of fraction; replace the first $\frac{1}{6}$ of your result with the first $\frac{1}{8}$ of patience; add the middle $\frac{1}{3}$ of accent (practice)

Today’s Challenge
Distribute the 18 Math Maze cards for week 25. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 202.

Student page 121 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 121 in the student book.

Answers for student page 121: 1. $\frac{1}{7}$ of 408; 1. $\frac{1}{7}$ of 360; 20. $\frac{1}{2}$ of 720; 3. $\frac{3}{4}$ of 36; 4. $\frac{3}{4}$ of 6400; 300. $\frac{2}{3}$ of 3600; 300. $\frac{3}{4}$ of 7200; 300. $\frac{3}{4}$ of 3500; 300.

8. neither 9. $\frac{3}{5}$ of 75; 2 10. $\frac{1}{3}$ of 7200; 100

Go Further
Student page 121 Ask students to make up their own Which is Greater problems to share with a friend.

Answers for student page 121: 11. Check students’ work. Solution methods will vary; the number is 120 and $\frac{1}{2}$ of the number is 80. 12. Check students’ work.

Assessment
Student self-assessment page 121 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students find many of these products using mental math?
Materials
Student page 122
Number cubes (1–6)
Chart paper and marking pen

Concepts and Handbook References
Simulate a probability experiment. (MOC 466)
Collect and organize data into a table. (MOC 285–286)

Background
Some companies put different prizes in their boxes to encourage people to buy their product. Today’s activity investigates that idea. Since students can’t purchase the product to see how many boxes they would need to buy in order to get all the prizes, they will do a simulation of that experiment. A simulation is a contrived experiment which models the real situation. In this case, students will roll a number cube until they get all six numbers, keeping track of how many rolls it took them to get those six numbers.

Get Started
Tell students that the Yummi Cereal Company has decided to randomly put an autographed picture of one of six famous baseball players into each of its Yummi Cereal boxes. The prizes will be distributed randomly in the boxes. Ask the students how many boxes they think they will need to buy in order to get the pictures of all six players. Many students will say you only need to buy six, but remind them that you might get multiple pictures of some players before you have one of each. Explain that statisticians often have to answer problems like this and that they do it with a simulation. Define simulation for the students.

Today’s Challenge
Student page 122 Pass out one 1–6 number cube to each pair of students. Tell them that they will roll the number cube to simulate buying boxes of cereal.

Answers for student page 122: Check students’ work to be sure they followed the rules.

Look Ahead
On chart paper, make a large copy of the table on student page 122.
Materials
Student page 123
Coin
Calculators

Concepts and Handbook References
Use mean, median, mode, and range to describe data. (MOC 271–272, 273–276)
Find the experimental probability of events. (MOC 466)

Background
Mean \( \frac{\text{sum of numbers}}{\text{number of numbers}} \)
Median middle number or mean of middle two numbers in an ordered set of data
Mode most frequent number or numbers

Get Started
Flip a coin six times and record the results on the board. Ask students to tell you the experimental probability of the coin landing heads up. Discuss how the experimental probability may differ from the theoretical probability, since it is based on an experiment.

Use a large copy of the table on student page 122 to make a master tally of results for that activity.

Go Further
Student page 123 Have the students work in pairs to find statistics for the class data collected during today’s Get Started activity. Two pairs of students may need to share a calculator.

Answers for student page 123: Check students’ work. Possible answers are provided.

Things we discovered:
• Rolling all six numbers in six rolls did not happen often (if ever).
• The results are not consistent, but the tallies group in the teens.

1–7. Check students’ work. Answers will vary depending on the data set. You would expect the mean to be around 14 to 15, with the median slightly less. The probability of rolling all six numbers in exactly six rolls should be very small (about 0.15), whereas the probability of rolling all six numbers in 15 or fewer rolls should be about 0.5.

Assessment
Student self-assessment page 123 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students comfortable with the difference between experimental and theoretical probability?
Materials
Student page 124
Blank paper (heavyweight if possible) or index cards

Concept and Handbook Reference
Compute using the order of operations.
(MOC 207–209)

Background
See page 41 for a review of order of operations.

Get Started
Remind students that the fraction bar is a grouping symbol. Warm up with these examples.
• What is $8 + (-2) \times 4$? (0)
• What is $-7 - 3^2 + 2$? ($-14$)*
• What is $(-3)^2 - 7 + 2$? (4)
• What is $\frac{25}{4} - 8$? ($-14$)
*Remind students that only the three is squared, not the subtraction symbol.

Today’s Challenge
Student page 124 Have students evaluate each expression in Today’s Challenge.

Answers for student page 124: 1. $-27$ 2. $-8$ 3. 2
4. 6 5. 4 6. $-27$ 7. 12 8. $-6$ 9. 0 10. 6 11. 3
12. 0 13. $-8$ 14. $-5$ 15. 4 16. 3 17. 2 18. $-6$
19. $-5$ 20. 12

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 20 small pieces of paper or 20 index cards. Have the students use one slip of paper or card to copy each problem on student page 124. They should not copy the answer.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a $4 \times 5$ array. The first player turns over any two cards. If the cards match (have the same answer when the problem is evaluated), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 124 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students use mental math and calculate correctly using the order of operations?
Materials
Student page 125
Graph paper

Concepts and Handbook References
Use combinations. (MOC 460)
Calculate the probability of simple and complementary events. (MOC 465, 470)
Construct a sample space as a means to answering probability and combinations questions.
(MOC 463–464)

Background
A combination is a group of items (or events).
Changing the order in which the items are listed does not create a new combination. For example, a committee of three people picked from a group of six people is the same committee regardless of who is picked first, second, or third. If all the outcomes of an event are equally likely, then the probability of a given combination occurring is the ratio of the number of ways the given items can be chosen to the total number of possible outcomes.

Constructing a sample space is a good strategy to use when calculating probabilities. Even when a test item does not require students to create a sample space, they should consider doing so on scrap paper, in the margin, or on the back of the test in order to clarify their thinking.

Get Started
Review how to find the probability of rolling a four on a 1–6 number cube. (2) Discuss how students could find the probability of a complementary event, such as not rolling a four, by subtracting the probability of rolling a four from one. (2) Discuss what a combination is.

Student page 125
Pass out graph paper. Have students work in pairs to make the sample space described at the top of student page 125. Make sure that every pair has the same sample space before they begin Today's Challenge.

Today's Challenge
Student page 125 Have students remain in pairs to complete Today's Challenge.


Go Further
On a separate sheet of paper, have each student write his or her name and create a data set and three problems similar to those on student page 125. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 125 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Can students create sample spaces to help them find the probabilities of events?
Materials
Student page 126
Math Maze Cards (Week 26 Activity 126)

Concept and Handbook Reference
Relate side length, perimeter, and area of polygons.
(MOC S60–S62)

Background
Perimeter of a square: 4s
Perimeter of a triangle: \( s_1 + s_2 + s_3 \)
Perimeter of a rectangle: \( 2l + 2w \)
Area of a triangle: \( \frac{1}{2}bh \)
Area of a square: \( s^2 \)
Area of a rectangle: \( lw \)
Area of a parallelogram: \( bh \)
Area of a rhombus: \( bh \)
Area of a trapezoid: \( \frac{1}{2}(b_1 + b_2) \)

Get Started
Review perimeter and area. If necessary, generate a list of formulas for perimeter and area. Keep it where students can see it and add to it.

Discuss why it is that the area of a figure does not, by itself, give you enough information to find its perimeter unless the figure is a square or other regular figure. (If the figure is a square, the length of a side is the square root of the area. Once you find side-length, you can find perimeter. For other plane figures, you need at least two lengths to generate the area and there may be many combinations of those two lengths that have the same product.)

Today’s Challenge
Distribute the 18 Math Maze cards for week 26. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 203.

Student page 126
When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 126 in the student book.

Answers for student page 126: 1. 28 in.; 48 sq. in.
2. 12 cm; 6 sq. cm 3. 24 in.; 28 sq. in. 4. 6 ft;
2.25 sq. ft 5. 32 cm; 52 sq. cm 6. 16.8 cm; 15 sq.
cm

Go Further
Student page 126 Help students think about the different rectangles that all have the same perimeter of 20 feet.

Answers for student page 126: 7–8. If the length and width columns are transposed, the answers are still correct.

<table>
<thead>
<tr>
<th>Perimeter (ft)</th>
<th>Length (ft)</th>
<th>Width (ft)</th>
<th>Area (sq. ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

9. It is a square.

Assessment
Student self-assessment page 126 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students reliably extrapolate missing information from given information? Can students recall area and perimeter formulas and use them correctly to solve problems?
Materials
Student page 127
Rulers
Chart paper and markers

Concepts and Handbook References
Use the Read and Make a Table and Look for a Pattern problem-solving strategies. (MOC 480, 484)
Use exponential notation. (MOC 071)

Get Started
Review the use of exponents to indicate repeated multiplication by the same factor.

Today’s Challenge
Student page 127 Pass out rulers. Draw a diagram of a large square on the board. Tell the students that the square represents Stage Zero. Now make a copy of stage zero and connect the trisection points of the sides of the square to form nine new squares. Shade in the center square. Tell them that this figure represents Stage One. Establish that, if the area of the large square is one square unit, then the area of each new, smaller square is \( \frac{1}{9} \) of a square unit. Since eight of these new squares are unshaded, their area is \( 8 \times \frac{1}{9} = \frac{8}{9} \) square units and the area of the remaining, shaded square is \( 1 - (8 \times \frac{1}{9}) \) square units. If entries in the table are made in this form, it will be easier for students to see a pattern. Work with students to copy stage one and connect the trisection points of each unshaded square to form eight subdivided squares, each of which has its center square shaded. Decide that the number of unshaded squares for that stage must be \( 8^2 \) before having them work in pairs on the rest of the activity.

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unshaded Squares</td>
<td>1</td>
<td>8</td>
<td>( 8^2 )</td>
<td>( 8^3 )</td>
</tr>
<tr>
<td>Shaded Squares</td>
<td>0</td>
<td>1</td>
<td>( 8 + 1 = 9 )</td>
<td>( 8^2 + 9 )</td>
</tr>
<tr>
<td>Area of Unshaded Squares (in square units)</td>
<td>1</td>
<td>( 8 \times \frac{1}{9} )</td>
<td>( 8^2 \times \frac{1}{9} )</td>
<td>( 8^3 \times \frac{1}{9} )</td>
</tr>
<tr>
<td>Area of Shaded Squares (in square units)</td>
<td>0</td>
<td>1 - ( (8 \times \frac{1}{9}) )</td>
<td>( 1 - (8^2 \times \frac{1}{9}) )</td>
<td>( 1 - (8^3 \times \frac{1}{9}) )</td>
</tr>
</tbody>
</table>

Look Ahead
On chart paper, draw the table on student page 127, extended through Stage 6. Tell students that they will analyze Stages 0–3 and predict the data for stages 4–6 next time.
Materials
Student page 128

Concepts and Handbook References
Use the Look for a Pattern, Make a Table, and Write an Equation problem-solving strategies. (MOC 484, 480, 482)
Use exponential notation. (MOC 071)

Background
A fractal is a mathematical object that is self-similar: each piece resembles the entire figure. Fractal-like objects are found in nature, such as ferns, lightning, waves in the ocean, and land formations in the Everglades in Florida. Repeated additions to or removals from a figure may form models of fractals. The Square Patterns activity involves splitting up squares into smaller squares, shading some according to a strict pattern. Each iteration is similar to the entire figure.

Get Started
Work with students to fill in the chart you made last time. (see page 127 for Stages 1–3) Have them work together to predict the number of shaded and unshaded squares in Stages Four, Five, and Six.

<table>
<thead>
<tr>
<th>Stage</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unshaded Squares</td>
<td>8^1</td>
<td>8^2</td>
<td>8^3</td>
</tr>
<tr>
<td>Shaded Squares</td>
<td>8^1 + 8^2 + 8^1 + 8^3</td>
<td>8^2 + 8^3 + 8^2 + 8^1 + 8^4</td>
<td>8^3 + 8^4 + 8^3 + 8^2 + 8^1 + 8^5</td>
</tr>
<tr>
<td>Area of Unshaded Squares (in square units)</td>
<td>8^1 \times \frac{1}{9^1}</td>
<td>8^2 \times \frac{1}{9^2}</td>
<td>8^3 \times \frac{1}{9^3}</td>
</tr>
<tr>
<td>Area of Shaded Squares (in square units)</td>
<td>1 - (8^4 \times \frac{1}{9^4})</td>
<td>1 - (8^5 \times \frac{1}{9^5})</td>
<td>1 - (8^6 \times \frac{1}{9^6})</td>
</tr>
</tbody>
</table>

Go Further
Student page 128 In order for students to recognize the formula for unshaded squares, they need to be familiar with the powers of eight: 1, 8, 64, 512, etc. The numbers of shaded squares are the sums of the powers of eight. (for stage n, there are \(8^n + 8^{n-1} + \ldots + 8^1\) squares) Another way to figure out the number of shaded squares is to add the number of unshaded and shaded squares from the previous stage. For example, the number of shaded squares in Stage Three is the sum of the number of unshaded squares in Stage Two (64) and the number of shaded squares in Stage Two (9). The number of shaded squares in Stage Three is 9 + 64 = 73.

Answers for student page 128: Check students’ work. Possible answers are provided.

Things we discovered:
• The number of unshaded squares in each stage is a power of eight.
• The number of shaded squares in each stage is the sum of all squares in the previous stage.
• The area of the unshaded squares in any stage, \(n\), is the number of unshaded squares divided by nine raised to the nth power.
• The area of the shaded squares in any stage is the area of unshaded squares subtracted from one.

1. \(8^\text{8th}\) square units 3. \(8^\text{8th} \times \frac{1}{9^\text{8th}}\) 4. \(8^\text{8th} \times \frac{1}{9^\text{8th}}\) square units

Assessment
Student self-assessment page 128 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students recognize that using exponential notation instead of simplifying an expression may help them see a pattern more readily?
Materials
Student page 129
Math Jumble activity poster and digit cards

Concept and Handbook Reference
Compute benchmark percents of whole numbers. (MOC 442–443)

Background
Thinking of benchmark percents as their equivalent fractions often turns computation into simple mental math. Common benchmark percents and their equivalent unit fraction are: $25\% = \frac{1}{4}$, $33\frac{1}{3}\% = \frac{1}{3}$, and $50\% = \frac{1}{2}$.

Get Started
Review using mental math to find benchmark percents using this example.

Find $50\%$, $33\frac{1}{3}\%$, and $25\%$ of 432.

Probe for these points.
- 432 is an even number and $50\% \left( \frac{1}{2} \right)$ of it is a whole number. One strategy is to think, $\frac{1}{2}$ of 400 is 200 and $\frac{1}{2}$ of 32 is 16 so $50\%$ of 432 is $200 + 16 = 216$.
- 432 is divisible by three (sum of digits is divisible by three) therefore $33\frac{1}{3}\%$ of it is a whole number. One strategy is to do mental short division.

\[
\frac{144}{3\overline{4}32}
\]

$33\frac{1}{3}\%$ of 432 is 144.
- 432 is divisible by four (last two digits are divisible by four) so $25\%$ of 432 is a whole number. One strategy is to think, $\frac{1}{4}$ of 400 is 100 and $\frac{1}{4}$ of 32 is 8 so $25\%$ of 432 is $100 + 8 = 108$.

Today’s Challenge
Use the 0–9 digit cards to construct this poster.

2 3 1 4
4 6 3 5
0 6 3 2
8 2 9 0

Explain that the object of today’s Math Jumble is to find three-digit numbers with whole-number products when multiplied by 25%, 33 1/3%, or 50%.

Numbers can be made by using any three adjoining digits (horizontally, vertically, and/or diagonally) on the poster. For example, the first three digits in the first column make the number 240. Record the facts students find in both word and symbol forms. For example, 25% of 240 is 60, $\frac{1}{4} \times 240 = 60$.

Possible numbers for 25%: 236 (59), 364 (91), 452 (113), 520 (130)

Possible numbers for 33 1/3%: 231 (77), 339 (113), 639 (213), 402 (134)

Possible numbers for 50%: 642 (321), 408 (204), 436 (218), 290 (145).

Student page 129 Have students use the Math Jumble on student page 129 to find more whole numbers that have whole-number products when multiplied by 25%, 33 1/3%, or 50%.

Answers for student page 129: Check students’ calculations. All multiples of four are correct answers for 25%. All multiples of three are correct answers for 33 1/3%. All even two-digit numbers are correct answers for 50%.

Go Further
Student page 199 Have students use the grid on the student page to create a Math Jumble to share with a friend.

Answers for student page 129: All multiples of four are correct answers for 25%. All multiples of three are correct answers for 33 1/3%. All even two-digit numbers are correct answers for 50%. Answers will vary. Check students’ calculations.

Assessment
Student self-assessment page 129 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students use mental math to find benchmark percents of a three-digit numbers?
Materials
Student page 130
Chart or overhead transparency of a blank fill-in grid
(save for future activities)
Calculators

Concepts and Handbook References
Compute surface area of prisms and cylinders.
(MOC 396, 401, 412)
Use a machine style fill-in grid. (MOC 532)

Background
\[ C(\text{circle}) = \pi d \]
\[ A(\text{circle}) = \pi r^2 \]
\[ A(\text{triangle}) = \frac{1}{2}bh \]
\[ A(\text{square}) = s^2 \]
\[ A(\text{rectangle}) = lw \]

Get Started
Review how to find surface area: find the sum of the areas of the faces. If necessary, make a master list of area formulas on the board for all to see.

Student page 130 Work with the class to find the surface area of the cylinder at the top of student page 130. (94.2 sq. cm) Make sure that everyone can successfully fill in the answer grid.

Today's Challenge
Student page 130 Have students sit in groups of three with access to calculators, but encourage them to work independently on Today's Challenge. Encourage them to draw another diagram if they need to for exercise 3.

Answers for student page 130: 1. 80.7 2. 62.8
3. 6 4. 471 5. Check students’ work. Advice might include ways to keep track of area of all the surfaces before adding them together.

Go Further
On a separate sheet of paper, have each student write his or her name, draw a diagram, and create a problem similar to those on student page 130. They should write the correct answer on the back. Have them share their problem with a friend, who will solve the problem, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students remember to add the bases of prisms and cylinders when finding surface area?
Materials
Student page 131
Math Maze Cards (Week 27 Activity 131)

Concept and Handbook Reference
Find a common percent of a dollar amount using mental math. (MOC 442-443)

Background
Students should be able to find the following common percents of a number using mental math:
10%, 25%, 33 1/3%, and 50%. For example, 10% of a number means 1/10 of the number so the decimal point will move one position to the left. (10% of $18.70 = \frac{1}{10} \times 18.70 = \$1.87). To find 25% of a number such as $24.60, students may find it easier to find 1/4 of $24 and 1/2 of $0.60 and then add their answers together to give $6.15.

Get Started
Review finding a percent of a number using mental math. Discuss with students that if they can compute 10% of a number, then they can also find 5% (1/2 of 10%), 15% (5% + 10%), and 20% (2 x 10%) of the same number. Try the following problems.

- How could you figure out 25% of $36.80?
  (25% = \frac{1}{4} \text{ of } 36.80 = \frac{1}{4} \times 36.80 = \$9.20)
- What is 50% of $62.50? (50% = \frac{1}{2} \text{ of } 62.50 = \$31.25)
- What is 33 1/3% of $15.45? (33 \frac{1}{3}\% = \frac{1}{3} \text{ of } 15 + \frac{1}{3} \text{ of } 0.45 = \$5.15)
- What is 5% of $48.20? (10% = \frac{1}{10} \text{ of } 48.20 = \$4.82 \text{ and } 5\% = \frac{1}{2} \text{ of } 10\% \text{ or } \$2.41)
- What is 15% of $48.20?
  (15\% \text{ of } 48.20 = 10\% \text{ of } 48.20 + 5\% \text{ of } 48.20 = \$4.82 + \$2.41 = \$7.23)

Today's Challenge
Distribute the 18 Math Maze cards for week 27. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have pencil and paper available for today's Math Maze.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card. You may wish to suggest that a pencil and scrap paper would be helpful for these problems.

The correct sequence of questions and answers is shown on page 204.

Student page 131 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 131 in the student book.

Answers for student page 131: 1. $1.84 2. $19.30 3. $7.10 4. $2.10 5. $4.25 6. $12.20 7. $13.30 8. $23.40 9. $15.00 10. $1.51

Go Further
Student page 131 Have students work in pairs on this section of the student page.

Answers for student page 131: 11. Check students’ work. One possible method is to round $48.35 to $48 and find 10% which is $4.80 and double it, so $9.60 is an estimate for the tip. 12. Check students’ work.

Assessment
Student self-assessment page 131 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students find many of these percents using mental math?
Materials
Student page 132
Colored squares

Concept and Handbook Reference
Extend geometric patterns. (MOC 484)

Get Started
Copy these figures on the board.

Figure 1  Figure 2  Figure 3

Ask students what relationships they see among the figures. Have students work in pairs and ask them to mark the diagram to illustrate their ideas.

Today's Challenge
Student page 132 Have the students work independently to complete the activity. Have colored squares available for those who need them.

Answers for student page 132:

1.

Figure 4  Figure 5

Figure 6

<table>
<thead>
<tr>
<th>Square Figure Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Squares</td>
<td>3</td>
<td>8</td>
<td>15</td>
<td>24</td>
<td>35</td>
<td>48</td>
</tr>
</tbody>
</table>

Look Ahead
Suggest that students who do not yet see any patterns in the table try a few more figures on their own before next time.
Materials
Student page 133

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Write an equation to describe a functional relationship. (MOC 205, 244)

Get Started
Work with students to recreate the filled-in table from activity 132 on the board.

Go Further
Student page 133 Once students have discovered a formula for the pattern, they should be able to apply that formula to find out how many squares are in any given figure. If students have trouble thinking about parts of the Figures, sketch this diagram for them.

You could extend this problem for some students by asking them what the formula would be if the row of squares in the arm of a figure was $n + 1$ squares long. \([(n + 1)(n + 3)]\)

Answers for student page 133: Check students’ work. Possible answers are provided.

Things we discovered:
• The $n$th figure has an $n$-by-$n$ square in the lower left hand corner.
• The $n$th figure has two rows of $n$ squares, one along the bottom and one on the left.
• The $n$th figure has $n(n + 2)$ squares in it.

1. multiply 17 by 19 to get 323 squares 2. a square
3. The Figure number tells the number of squares in each arm. 4. number of squares = $n(n + 2)$ or $n^2 + 2n$ 5. $143 = n(n + 2)$; $n = 11$

Assessment
Student self-assessment page 133 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students write an equation to describe a pattern?
Materials
Student page 134
Slips of paper containing numbers from one through 36.

Concept and Handbook Reference
Review measurement facts. (MOC 536)

Get Started
Make a list of customary measurement facts for length, time, capacity, and so on. Once the list has been made with student input, ask questions like these.

• How many seconds are in \( \frac{3}{4} \) of a minute? (45)
• How many inches are in \( \frac{3}{4} \) foot? (8)
• How many ounces are in \( 1 \frac{1}{4} \) pounds? (20)

Today's Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with one of the numbers one through 36 on it. You do not need to use all of the numbers, but be sure that one student receives the number 27, since that number will be the “Fantastic Finalist.”

Have all students hold their numbers and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
• If your number is divisible by the number of inches in half of a foot, sit down. (6, 12, 18, 24, 30, 36)

• If your number is greater than or equal to the number of ounces in \( 1 \frac{1}{4} \) pounds, sit down. (28, 29, 31, 32, 33, 34, 35)

• If your number is less than the number of minutes in \( \frac{1}{3} \) of an hour, sit down. (1–5, 7–11)

• If your number is less than the number of quarts in six gallons, sit down. (13–17, 19–23)

• If your number is less than the number of inches in \( \frac{3}{4} \) of a yard, sit down. (25, 26)

At this point, only the student holding the number 27 should still be standing. That student is the Fantastic Finalist.”

Go Further
Student page 134 Have students complete the activity on the student page and create a “Fantastic Finalist” activity for a friend to solve.

Answers for student page 134: 1–5.

<table>
<thead>
<tr>
<th>1&lt;sup&gt;st&lt;/sup&gt;</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt;</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt;</th>
<th>4&lt;sup&gt;th&lt;/sup&gt;</th>
<th>5&lt;sup&gt;th&lt;/sup&gt;</th>
<th>6&lt;sup&gt;th&lt;/sup&gt;</th>
<th>7&lt;sup&gt;th&lt;/sup&gt;</th>
<th>8&lt;sup&gt;th&lt;/sup&gt;</th>
<th>9&lt;sup&gt;th&lt;/sup&gt;</th>
<th>10&lt;sup&gt;th&lt;/sup&gt;</th>
<th>11&lt;sup&gt;th&lt;/sup&gt;</th>
<th>12&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
</tbody>
</table>

6. 17 7. Check students’ work.

Assessment
Student self-assessment page 134 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students recall basic measurement facts and find fractional parts of units?
Materials
Student page 135

Concepts and Handbook References
Read and interpret circle graphs. (MOC 296)
Estimate with percents to choose from among four answer choices. (MOC 442, 528)

Get Started
Take a quick survey of the kinds of shoes students are wearing. List each shoe-type and tally the number of students wearing each type.

Example:
- Sneakers
- Sandals
- Boots
- Other

Ask how the data could be graphed. Suggestions should include bar graph, pictograph, and circle graph, since they can effectively display parts of a whole. Discuss the fact that there is no mean or median, since the data represent things, not numbers. Note that a stem-and-leaf plot, a box-and-whisker plot, and a line graph would not be informative displays of the data.

Student page 135 Ask students to study the circle graph at the top of student page 135. Tell them that they should estimate to decide which answer choice is best for the question. After a few minutes, discuss their reasons for accepting or rejecting each answer choice. (Estimate: 192 flags, 10% would be about 19 flags, 5% would be fewer than 10 flags, so 7.3% is between nine and 19. One less would be between eight and 18. The only good choice is D, None of these.)

Today’s Challenge
Student page 135 Do not provide calculators. Tell students that they should estimate to choose the correct answers for Today’s Challenge.


Go Further
On a separate sheet of paper, have each student write his or her name and create a problem similar to those on student page 135. Students may use the graph on student page 135 or create a new one. They should write the correct answer on the back. Have them share their problem with a friend, who will solve the problem, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 135 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Can students estimate with percents?
Materials
Student page 136
Math Maze Cards (Week 28 Activity 136)

Concept and Handbook Reference
Use exponents. (MOC 071)

Background
To square a number is to raise it to the second power. To cube a number is to raise it to the third power. If a number is negative, its square (or any even power) will be positive ($(-4 \times -4 = 16)$) and its cube (or any odd power) will be negative ($(-4 \times -4 \times -4 = 16 \times -4 = -64)$). Raising a fraction to a power involves multiplying the fraction by itself.

Get Started
Ask students what four squared is. A common mistake is to multiply four by two instead of four by four. Ask what two cubed is. Extend the discussion to raising fractions and integers to a power. Try the following examples.

- What is $\frac{1}{2}$ cubed? ($\left(\frac{1}{2}\right)^3 = \frac{1}{8}$)
- What is the square of $-5$? ($-5 \times -5 = 25$)
- What is $-2$ raised to the fifth power? ($-2 \times -2 \times -2 \times -2 \times -2 = -32$)

Today’s Challenge
Distribute the 18 Math Maze cards for week 28. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 205.

Student page 136 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 136 in the student book.

Answers for student page 136: 1. 25 2. -8 3. 81 4. $\frac{1}{4}$ 5. 27 6. -81 7. 0.01 8. -27 9. 64 10. $-\frac{1}{125}$ 11. 32 12. -1 13. 100 14. THE POWER OF NUMBERS

Go Further
Student page 136 Have students complete this section of the student page.

Answers for student page 136: 15. 16, 9, 4, 1, 0, 1, 4, 9, 16 16. -64, -27, -8, -1, 0, 1, 8, 27, 64 17. Check students’ work. Raising a positive number to a power results in a positive answer; squaring a negative number results in a positive answer; cubing a negative number results in a negative number; there is symmetry around zero—the absolute values of powers of opposites are equal.

Assessment
Student self-assessment page 136 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students able to raise integers and signed unit fractions to a power?
Materials
Student page 137
Colored squares
Paper bags

Concepts and Handbook References
Collect and organize data. (MOC 263–269)
Find experimental probability. (MOC 466)

Get Started
Place two squares of one color and two squares of another color in a paper bag and tell students you are going to randomly pick two squares at the same time. Ask them if they think the two are more likely to be the same color, or different colors. Ask students to explain their reasoning. Replace the squares. Do the experiment several times and record the data on the board. After doing this 10 times, ask them what they think now. (This is not a fair game; in fact, the probability that the two squares will be different is $\frac{5}{8}$, twice the probability that they will be the same. Do not share this fact with students yet.)

Today’s Challenge
Student page 137 Have the students work in pairs.
Pass out five colored squares, three of one color and two of a second color, to each pair of students.

Answers for student page 137: 1–2. The data in the table will vary, but the Different category should have a probability of about $\frac{2}{3}$ and the Same category should have a probability of about $\frac{1}{3}$. 3–4. The data will vary, but the Same category should have about the same probability as the Different category.

Look Ahead
If students do not trust their results, suggest that they play more games in each category (3-2 or 3-1) before next time.
Materials
Student page 138

Concepts and Handbook References
Create a sample space. (MOC 463, 464)
Find the probability of simple events. (MOC 465)

Background
While experimental probability is based on the results of trials, theoretical probability is based on the number of possible outcomes and their likelihood of occurring.

Get Started
Draw this table on the board and have students (one from each pair) record the results of the games they played for activity 137.

<table>
<thead>
<tr>
<th>3-2 Game</th>
<th>Pair 1</th>
<th>Pair 2</th>
<th>Pair 3</th>
<th>...</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw and complete a second table for the 3-1 game. Fill in the total columns and leave the tables up while students work on today’s activity.

Go Further
Student page 138 Show students how to use subscript notation. Each square in the bag is unique, so each gets a name. R₁ and R₂ can represent the red squares and B₁, B₂ the blue squares. R₁ B₁ is a different outcome than R₂ B₁ for purposes of the sample space, even though you may see no difference. Since you draw one square and then, without replacing that square, draw a second square, R₁ B₂ is a different outcome than B₂ R₁ for purposes of the sample space.

Answers for student page 138: Check students’ work. Possible answers are provided.

Things we discovered:
• Two squares of two different colors do not produce a fair game.
• The game with three squares of one color and one of a different color seems to be fair.

1. Second Pick

<table>
<thead>
<tr>
<th>3-2 Game</th>
<th>Red₁</th>
<th>Red₂</th>
<th>Blue₁</th>
<th>Blue₂</th>
<th>Blue₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red₁</td>
<td>R₁ R₂</td>
<td>R₁ B₁</td>
<td>R₁ B₂</td>
<td>R₁ B₃</td>
<td></td>
</tr>
<tr>
<td>Red₂</td>
<td>R₂ R₁</td>
<td>B₂ B₁</td>
<td>B₂ B₂</td>
<td>B₂ B₃</td>
<td></td>
</tr>
<tr>
<td>Blue₁</td>
<td>B₁ R₁</td>
<td>B₁ R₂</td>
<td>B₁ B₂</td>
<td>B₁ B₃</td>
<td></td>
</tr>
<tr>
<td>Blue₂</td>
<td>B₂ R₁</td>
<td>B₂ R₂</td>
<td>B₂ B₁</td>
<td>B₂ B₃</td>
<td></td>
</tr>
<tr>
<td>Blue₃</td>
<td>B₃ R₁</td>
<td>B₃ R₂</td>
<td>B₃ B₁</td>
<td>B₃ B₂</td>
<td></td>
</tr>
</tbody>
</table>

2. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{3}{3}$. This is not a fair game, since the probabilities are not equal.

4. Second Pick

<table>
<thead>
<tr>
<th>3-1 Game</th>
<th>Red₁</th>
<th>Red₂</th>
<th>Red₃</th>
<th>Blue₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red₁</td>
<td>R₁ R₂</td>
<td>R₁ R₃</td>
<td>R₁ B₁</td>
<td></td>
</tr>
<tr>
<td>Red₂</td>
<td>R₂ R₁</td>
<td>R₂ R₃</td>
<td>R₂ B₁</td>
<td></td>
</tr>
<tr>
<td>Red₃</td>
<td>R₃ R₁</td>
<td>R₃ R₂</td>
<td>R₃ B₁</td>
<td></td>
</tr>
<tr>
<td>Blue₁</td>
<td>B₁ R₁</td>
<td>B₁ R₂</td>
<td>B₁ R₃</td>
<td></td>
</tr>
</tbody>
</table>

5. $\frac{1}{2}$. 6. This is a fair game since the probability of colors being the same is equal to the probability of colors being different.

Assessment
Student self-assessment page 138 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students recognize the need to differentiate between the two red squares when making a sample space with equally likely outcomes?
Materials
Student page 139

Concept and Handbook References
Add and subtract rational numbers.  
(MOC 108, 136)

Background
See page 65 for a review of integer operations.

Get Started
Talk about situations in which students have added signed numbers. Examples might include receiving a paycheck of $24.75 and spending $10.50  
(24.75 + (-10.50) = 14.25); the temperature dropping 4°C and then rising 6°C  
(-4 + 6 = 2); gaining 3 yards on a play followed by losing 5.5 yards on the next play  
(3 + (-5.5) = -2.5). Have students explain how to add two positives, two negatives, and one  
positive and one negative. Then, evaluate and discuss these expressions.

- What is 6.8 + (-4.1)? (2.7)
- What is 3.2 + (-5.6)? (-2.4)

Student page 139 Have students complete the table in the Get Started section.

Answers for student page 139:

<table>
<thead>
<tr>
<th></th>
<th>3.1</th>
<th>5.6</th>
<th>6.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.4</td>
<td>0.7</td>
<td>3.2</td>
<td>4.4</td>
</tr>
<tr>
<td>-4.7</td>
<td>-1.6</td>
<td>0.9</td>
<td>2.1</td>
</tr>
<tr>
<td>-8.3</td>
<td>-5.2</td>
<td>-2.7</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

Today’s Challenge
Explain that today the class will be playing a game called “Who Wants to Be the Top Scorer?” Have each student take a blank sheet of paper and write an addition equation in which each addend is from -9.0 through 9.0, then ask them to number their papers from 1 through 5.

As you ask each question, have students look at their sums and answer the question. Yes answers will score points.

1. Do the two addends have opposite signs? If yes, score 10 points.
2. Are both addends negative? If yes, score 9 points.
3. Is the sum between -2 and 2? If yes, score 8 points.
4. Is the sum zero? If yes, score 5 points.
5. Do the two addends have opposite signs with a sum less than -6? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student write the addition problem on the board and explain how he or she scored the points. If no student earned 25 points, discuss ways to earn this maximum score. (Possible answer: 1 + (-8) = -7)

Go Further
Student page 139 Have students solve the riddles, complete the riddle, and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 139: 2. 1.5  3. 8.3
4–5. Check students’ riddles.

Assessment
Student self-assessment page 139 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students add and subtract rational numbers using mental math?
Materials
Student page 140

Concepts and Handbook References
Read, interpret, and compare circle graphs, bar graphs, and pictographs. (MOC 292, 294, 296)
Provide clear, complete short answers and explanations. (MOC 531)

Get Started
Tell students that today they will consider various graphs and decide under what circumstances one kind of graph is more useful than another.

Student page 140 Have students work in pairs to study the graphs and answer the questions at the top of student page 140. After about 10 minutes, ask volunteers to suggest answers. Discuss thoroughly before assigning Today's Challenge. (A. circle graph B. bar graph C. Student opinion may split on this question. Good reasons are more important than a specific choice.

Today's Challenge
Student page 140 Keep students in pairs for Today's Challenge.

Answers for student page 140: Check students' work. Possible answers are provided. 1. It is easier to get a quick approximation of the actual numbers involved. 2. It is difficult to say what portion is represented by a partial figure. 3. Use circle graphs to get a good sense of the whole picture. Use bar graphs to show comparisons of parts to each other. Use pictographs instead of bar graphs when close approximations of numbers are not so important.

Go Further
On a separate sheet of paper, have each student write his or her name and create a problem similar to those on student page 140. Students may use the graphs on student page 140 or create their own. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 140 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Can students select the type of graph that would best reflect the information they want conveyed?
Materials
Student page 141
Math Maze Cards (Week 29 Activity 141)

Concept and Handbook Reference
Use order of operations with integers. (MOC 207–209)

Background
See page 41 for a review of order of operations.

Get Started
Begin by asking these questions.
• What operation is performed first in $23 + 7 \times 8$? (multiplication)
• What is $35 - 4 \times -7$? (63)
• What is $-48 \div 8 \times 3$? -18
• What is $-10 + 6 \times 9$? (44)

Today’s Challenge
Distribute the 18 Math Maze cards for week 29. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 206.

Student page 141 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 141 in the student book.

Answers for student page 141: 1. 17  2. 78  3. 24
4. 89  5. 9  6. 0  7. 10  8. -56  9. 28  10. 63

Go Further
Student page 141 Have students complete this section of the student page.

Answers for student page 141:
17. $(18 + -9) \div 3 + -20 = -17$
18. $-9 \times (4 - 2) = -18$
19. $(6 + 3) \times (8 - 3) = 45$
20. Check students’ work.

Assessment
Student self-assessment page 141 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Are students’ mental-math skills sharp enough to solve these problems without paper and pencil? Do students correctly perform integer operations?
Materials
Student page 142
Colored squares
Paper bags

Concept and Handbook Reference
Solve probability problems involving dependent events. (MOC 469)

Background
See student page 137 for Same or Different game rules.

Get Started
Remind students of the Same or Different game. Tell them that they will be playing with nine squares of two colors distributed 5-4, 6-3, 7-2, or 8-1. Ask students whether they think any of these games will be fair and have them explain their reasoning. Draw this table on the board and discuss how students can record their results in it without disrupting the group.

<table>
<thead>
<tr>
<th></th>
<th>5-4 Game</th>
<th>6-3 Game</th>
<th>7-2 Game</th>
<th>8-1 Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Today's Challenge
Student page 142 Have students work in pairs and assign at least two groups of students to collect data about each of the four cases. Pass out colored squares that match the assigned games (each pair plays only one of the four games). Have students record their data in the class table, so that it can be used to complete student page 142.

Answers for student page 142: 1. Accept any prediction. Answers will vary based on the data 2. the 6-3 game is fair. 3. increases 4. 7, 6

Look Ahead
Encourage students to play before next time some of the games they did not play today.
Materials
Student page 143

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy.
(MOC 484)
Find the probability of dependent events.
(MOC 469)

Background
Two events are dependent if the outcome of one event affects the probability of the second event. The probability of two dependent outcomes is the product of the probability of each outcome. For the Same or Different game, the probability of two same colors is the sum of the probabilities of two of one and two of the other.

Get Started
Pose the following problem to students: If I have three red squares and two blue squares in a bag, what is the probability of randomly selecting a red square? (3) Now if I drew another square without replacing the first one, what would be the probability it would be red? (because two red squares and two blue squares remain in the bag after the first pick)

Discuss probability of dependent events. If there are n items to pick from, then for each first pick there are n - 1 possible second picks. If there are a ways to pick the item you want, then there are a - 1 ways to pick it on the second round. Try a few examples using the Same or Different game rules.

• 4 red and 1 blue
Find the probability of picking red, then blue.
Pick 1: 4
Pick 2: 1
Red, then blue → × =

• 3 red and 2 blue
Find the probability of picking red, then red.
Pick 1: 3
Pick 2: 2
Red, then red → × =

• 3 red and 2 blue
Find the probability of picking both the same color.
Red, then red = (see previous example)
Blue, then blue → × =
Both the same →

Go Further
Student page 143 Have students work in pairs to complete student page 143.

Answers for student page 143: 1. 2. 3. 4.

Assessment
Student self-assessment page 143 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students compute probabilities of dependent events?
Materials
Student page 144
Blank paper (heavyweight if possible) or index cards

Concepts and Handbook References
Use mental math to solve two-step equations.
(MOC 241)
Use inverse operations. (MOC 224)

Background
To solve an equation for a variable means to find the value of the variable that makes the equation true. To solve an equation, simplify each side by combining like terms, then perform the same operation on both sides of the equation. Follow order of operations in reverse until the variable is by itself on one side of the equation.

Get Started
Review how to use inverse operations to solve equations. Addition and subtraction, and multiplication and division, are inverse operations. Ask students to explain their steps in solving these equations.
- Solve for $x$: $5x - 4 = 11$ (3; add four to both sides, then divide both sides by five)
- Solve for $x$: $-2x + 7 = 19$ (-; subtract seven from both sides, then divide both sides by negative two)
- Solve for $x$: $6x - 4x = -10$ (-; subtract 4x from 6x, then divide both sides by two)

Today's Challenge
Student page 144 Have students complete the table on student page 144. They should use inverse operations to solve each equation for $x$.

Answers for student page 144: 1. 4 2. 6 3. -2 4. 1 5. 8 6. 3 7. 4 8. 10 9. -2 10. 5 11. 3 12. 9 13. 4 14. 5 15. 10 16. 8 17. 6 18. -4 19. 9 20. 1

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 20 small pieces of paper or 20 index cards. Have the students use one slip of paper or card to copy the equation from each problem on student page 144. Students should not copy the answer to the problem.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a $4 \times 5$ array. The first player turns over any two cards. If the cards match (have the same value for $x$ when the equations are solved), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 144 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students use inverse operations to solve two-step equations?
Materials
Student page 145
Calculators

Concepts and Handbook References
Find volume of common three-dimensional figures. (MOC 402, 413)
Use the Draw a Diagram problem-solving strategy. (MOC 483)
Pay particular attention to units when selecting from among answer choices. (MOC 529)

Background
Volume of rectangular prism \( V = \ell \times w \times h \)
Volume of cylinder \( V = \pi r^2 h \)

Get Started
Review formulas for volume of a rectangular prism and a cylinder. Then, draw this diagram on the board.

\[ 8\sqrt{2} \text{ cm} \]

Ask students to tell you everything they know or can find out about this figure. Probe for these points.
- The figure is a square.
- The diagonal divides the square into two congruent right triangles.
- The length of a side can be computed. The Pythagorean theorem states that in a right triangle with legs \( a \) and \( b \) and hypotenuse \( c \) units long, \( a^2 + b^2 = c^2 \). Here, \( c = 8\sqrt{2} \) and \( a = b \).

\[
\begin{align*}
\ell^2 + \ell^2 &= (8\sqrt{2})^2 \\
2\ell^2 &= 8 \times \sqrt{2} \times 8 \times \sqrt{2} \\
2\ell^2 &= 8^2 \times (\sqrt{2})^2 \\
2\ell^2 &= 64 \times 2 \\
2\ell^2 &= 128 \\
\ell^2 &= \frac{128}{2} \\
\ell^2 &= 64 \\
\ell &= 8
\end{align*}
\]

Student page 145 Encourage students to use the side length you just derived as they solve the Get Started problem. Ask them why drawing their own diagrams would help if they had not already figured out the length of the side of a square with a diagonal \( 8\sqrt{2} \) centimeters long. (A is correct; a diagram of the base by itself would reveal a square with the given diagonal.)

Today's Challenge
Student page 145 Have students work in small groups with access to calculators. Encourage them to draw diagrams to help clarify each problem, and to show their work. Tell them to be very careful with units.


Go Further
On a separate sheet of paper, have each student write his or her name, draw a diagram, and create two problems similar to those on student page 145. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 145 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students draw a diagram to help clarify their thinking?
Math Maze

Materials
Student page 146
Math Maze Cards (Week 30 Activity 146)

Concept and Handbook Reference
Evaluate expressions with integers. (MOC 206)

Background
An algebraic expression is a collection of numbers, variables, operations, and grouping symbols.

Examples:
• \(-6x + 15\) −6 times \(x\) plus 15
• \(x ÷ (-7)\) \(x\) divided by \(-7\)
• \(4(x - 6)\) four times the difference of \(x\) and six

The second expression, \(x ÷ (-7)\), could also be written \(\frac{x}{-7}\).

To evaluate an expression for a specific value of the variable, you replace the variable by the number, then simplify the numerical expression.

Example:
What is the value of \(-6x + 15\) when \(x = 4\)?
Evaluate: \(-6x + 15 = -6(4) + 15 = -24 + 15 = -9\)

Get Started
Talk with students about expressions and how to evaluate them, then do a few practice examples.
• What is the value of \(6x - 4\) when \(x = -9\)? (-58)
• What is the value of \(x^2 + 8\) when \(x = 57\)? (33)
• What is the value of \(\frac{x}{4} + 3\) when \(x = -8\)? (5)

Remind students about the order of operations and raising a number to a power. In the expression \(4 + 2x\), there are two operations involved, addition and multiplication. When the variable is replaced by a number, it is multiplied by two before it is added to four.

Today’s Challenge
Distribute the 18 Math Maze cards for week 30. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 206.

Student page 146 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 146 in the student book.

Answers for student page 146: 1. -15 2. 27 3. -31 4. 30 5. 13 6. 1 7. -35 8. 10 9. 8 10. 52

Go Further
Student page 146 Encourage students to look at the results of evaluating an expression for different integer values of the variable.

Answers for student page 146: 11. 8, 5, 2, -1, -4, -7, -10, -3, -16 12. -5, -3, -1, 1, 3, 5, 7, 9, 11 13. Check students’ work. Possible answers are provided. Values for \(-3x - 4\) for consecutive values of \(x\) decrease by three. Values for \(2x + 3\) for consecutive values of \(x\) increase by two. The answers to exercise 11 alternate, even, odd, even, odd, and so on. The absolute values of all of the answers to exercise 12 are odd.

Assessment
Student self-assessment page 146 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students evaluate an expression when the value of the variable is given?
**Materials**
Student page 147  
Sugar cubes (optional)  
Chart paper and markers

**Concept and Handbook Reference**
Use the Look for a Pattern problem-solving strategy. (MOC 484)

**Get Started**
Draw this diagram.

![Three-Cube Diagram]

Ask the class how many unit cubes could be made from this three-cube. (27) Tell the class that the three-cube was painted on the outside, then cut into 27 unit cubes. Ask how many cubes have been painted on exactly three faces. (8) Two faces. (12) One face. (6) No faces. (1)

**Today’s Challenge**
**Student page 147** Have the students work independently to fill in the table on student page 147. Some students may need to construct the cubes to help them fill out the table. For this, you might offer sugar cubes and markers.

**Answers for student page 147:**

<table>
<thead>
<tr>
<th>Cubes</th>
<th>Two-Cube</th>
<th>Three-Cube</th>
<th>Four-Cube</th>
<th>Five-Cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three Faces Painted</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Two Faces Painted</td>
<td>0</td>
<td>12</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>One Face Painted</td>
<td>0</td>
<td>6</td>
<td>24</td>
<td>54</td>
</tr>
<tr>
<td>Zero Faces Painted</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>27</td>
</tr>
</tbody>
</table>

2. three faces painted: 8; two faces painted: 48; one face painted: 96; no faces painted: 64

**Look Ahead**
On chart paper, copy the table on student page 147. If students have not noticed any patterns yet, encourage them to use sugar cubes to create six- and seven-cubes.
Materials
Student page 148
Sugar cubes and colored pens (optional)
Calculators

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy.
(MOC 484)
Write an equation to describe a functional relationship. (MOC 205, 244)

Get Started
With students, fill in the chart-sized table of results for activity 147. (See answers for student page 147.)

Go Further
Student page 148 Have students work in pairs or small groups with calculators available. If they struggle, you might allow them to construct sugar-cube models, marking the cubes with colored pens.

Answers for student page 148: Check students’ work. Possible answers are provided.

Things we discovered:
• The three faces-painted unit cubes are on the corners of their cubes.
• All cubes with a length of two or more units have eight unit-cubes with three painted faces.
• The two-faces-painted unit cubes are on the edges.
• The number of unit cubes with one face painted is a multiple of six.
• The one-face-painted unit cubes are in the center of each face.
• The zero-faces-painted unit cubes are in the interior of their cubes.
• The number of cubes painted with zero faces is a whole number cubed.

1. 8 8 2. It increases by 12. 3. 96; \( F_2 = 12(n - 2) \)
4. You get a square number. 5. 384;
\( F_1 = 6(n - 2) \) 6. 512; \( F_0 = (n - 2)^2 \) 7. 8; 216;
1944; 5832

Assessment
Student self-assessment page 148 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Do students use their sense of patterns (as in, the cube inside a cube has sides \( n - 2 \) units long) to write equations?
Materials
Student page 149
Math Jumble activity poster and digit cards

Concept and Handbook Reference
Use the Distributive Property. (MOC 220)

Background
The Distributive Property states that for any real numbers \( a, b, \) and \( c, \) \( a(b + c) = ab + ac. \) Since any subtraction expression can be rewritten as an addition expression, the Distributive Property also works for subtraction: \( a(b - c) = ab - ac. \)

Get Started
Review the Distributive Property. The property involves two operations, multiplication and addition (or subtraction). Ask students to expand and evaluate these expressions.
- \( 6(4 + 3) = 6 \times 4 + 6 \times 3 = 42 \)
- \( 8(5 + 2) = 8 \times 5 + 8 \times 2 = 56 \)
- \( 4(7 + 6) = 4 \times 7 + 4 \times 6 = 52 \)
- \( 9(8 - 3) = 9 \times 8 - 9 \times 3 = 45 \)

Today's Challenge
Use the 0–9 digit cards to construct this poster.

```
  6 4 3 7
  2 6 8 2
  5 8 4 6
  3 4 9 8
```

Explain that the object of today's Math Jumble is to find a string of three digits that could be written in the form \( a(b + c) \) or \( a(b - c) \) such that the value is between 40 and 60. Remind them that when the word \textit{between} is used, the boundary numbers are not included.

Strings of numbers are made by connecting adjoining digits (horizontally, vertically, and/or diagonally) on the poster. For example, the first three digits in the first column can be used to make the expression \( 6(2 + 5). \) For each expression, students should write the expanded version and give the value.

Possible strings: \( 6(4 + 3) = 6 \times 4 + 6 \times 3 = 42; \)
\( 7(2 + 6) = 7 \times 2 + 7 \times 6 = 56; \)
\( 5(6 + 4) = 5 \times 6 + 5 \times 4 = 50; \)
\( 9(8 - 3) = 9 \times 8 - 9 \times 3 = 45 \)

Student page 149 Have students use the Math Jumble on student page 149 to find more strings of digits that could be used to write expressions involving the Distributive Property.

Answers for student page 149: 1–4. Check students' work. Possible answers are provided.
- \( 8(3 + 4) = 8 \times 3 + 8 \times 4 = 56; \)
- \( 4(7 + 5) = 4 \times 7 + 4 \times 5 = 48; \)
- \( 9(5 + 1) = 9 \times 5 + 9 \times 1 = 54; \)
- \( 9(8 - 2) = 9 \times 8 - 9 \times 2 = 54 \)

Go Further
Student page 149 Have students use the grid on the student page to create a Math Jumble to share with a friend.

Answers for student page 149: Grids and patterns will vary. Check students' work.

Assessment
Student self-assessment page 149 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students correctly evaluate an expression using the Distributive Property?
Materials
Student page 150
Calculators

Concepts and Handbook References
Check solutions to equations by substituting, using a calculator if appropriate. (MOC 242)
Compute with fractions, decimals, and integers. (MOC 102–113, 131–141, 158–170, 184–199)

Background
To enter a negative number into most calculators, enter the absolute value of the number, then use the change-sign key to display the negative sign.

Example: Multiply $-75$ by 2.5

\[
\begin{array}{c}
7 \\ 5 \\ + \\ 2 \\ .5 \\
\end{array} = \begin{array}{c}
-187.5 \\
\end{array}
\]

To compute with fractions using the KC-129 calculator that came with your kit, use the \textbf{a}/\text{b}/ key.

Example: Subtract \(4\frac{1}{2}\) from 6

\[
\begin{array}{c}
6 \\ - \\ 4 \\ \text{a}/\text{b} \\ 2 \\ \text{a}/\text{b} \\ 5 \\
\end{array} = \begin{array}{c}
1.35 \\
1\frac{1}{2} \\
\end{array}
\]

Get Started
Review order of operations if you need to. (Refer to page 41.) Show students how to enter a negative number into the calculator using the change-sign key and review how to use the calculator to compute with fractions.

Discuss checking equations. When students solve an equation they should substitute the solution back into the original equation as a means of checking to be sure that the solution satisfies the equation (makes it true). Write this equation on the board, \(3(2x - 5) + 4 = 4x + 1\), and ask whether \(x = 6\) is the correct solution. Have students substitute six for \(x\) in the equation and simplify to see whether it works. (It does.) Have a volunteer come to the board to show his or her work.

In a test situation, when computation is messy and computation is not the skill being tested, calculators are often allowed. Students need to be able to decide when a calculator will actually save time. Ask whether a calculator will help check whether \(x = 4\) is a solution for \(\frac{6x - 6}{4} + 2\frac{1}{2} = 2x - 3\frac{3}{4}\). (Probably not; 4 is not a solution) Have someone volunteer to show his or her work on the board. Solve the equation with the students \((x = 7)\) and have them substitute their solution back into the equation to be sure it works. (It does.)

Student page 150 Have the students work in small groups with access to calculators as they try the problem at the top of student page 150. Then ask someone to volunteer to do the problem on the board. \((y = 4.9, y = 3.4;\) The calculator may have been useful to test the values in the equation.)

Today’s Challenge
Student page 150 Encourage students to work independently on student page 150 while they remain seated in small groups sharing calculators.

Answers for student page 150: 1. Yes \ 2. No; \(m = 4.7\) \ 3. Yes \ 4. No; \(p = -2.3\) \ 5. Yes \ 6. Check students’ work. \ 7. Check students’ advice. They should include having students substitute the answer back into the original equation, and using a calculator if the computation is messy and time consuming.

Go Further
On a separate sheet of paper, have each student write his or her name and create two problems similar to those on student page 150. They should write the correct answers on the back. Have them share their problems with a friend, who will solve the problems, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 150 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Do students substitute a value for the variable and simplify the equation to verify a solution?
Materials
Student page 151
Math Maze Cards (Week 31 Activity 151)

Concept and Handbook Reference
Find percent increase and percent decrease.
(MOC 447–448)

Background
When amount of change is the absolute value of
the difference between original amount and new
amount, Percent Change = \( \frac{\text{amount of change}}{\text{original amount}} \).

Get Started
Students should be able to find benchmark per-
cents, such as 10%, 25%, and 50%, of a number.
With this skill they can find the result of increasing
or decreasing a given number by a specific percent.

Examples:
• To find a 50% increase of 150, first compute 50% of
150 (75) and add it on to 150 (225).

• To find a 200% increase of 150, take 100% of 150
(150), double it for 200% (300) and add it on to
150 (450).

• To find the percent decrease when original price is
$40 and discounted price is $32, find amount of
change \( (40 - 32 = 8) \), divide amount of change
by original price \( (8 \div 40 = 0.2) \) and write that as
a percent: 20%.

Today's Challenge
Distribute the 18 Math Maze cards for week 31.
Each student should receive at least one card, but
since all cards need to be distributed, some students
may need to get more than one card. Use the cards
to play the Math Maze game.

Instructions for playing Math Maze Ask students to
look at their cards. Ask one student to read the
question that is written on his or her card. Next ask,
"Who has the card with the answer to the question
just read?" Ask that student to read the answer, and
then read the question on his or her card. Play con-
tinues until all questions have been answered. The
last answer to be read should be the answer on the
first student's card.

The correct sequence of questions and answers is
shown on page 207.

Student page 151 When the group has finished
playing the game, have students open their books
and complete the Today's Challenge activity.

Answers for student page 151: 1. $50; 20%
increase 2. $200; 50% decrease 3. $150; 25%
increase 4. $300; 300% increase 5. $375; 75%
decrease 6. $150; 75% increase 7. $20; 25%
decrease 8. $70; 20% decrease 9. $90; 25%
decrease 10. $225; 150% increase

Go Further
Student page 151 You may wish to have students
work in pairs on this section of the student page.

Answers for student page 151: 11. The amount of
change is $2.40; divide the amount of change by
the original amount ($40) and the quotient is 0.06
which is 6%. 12. The amount of change is $19.20;
divide the amount of change by the original
amount ($96) and the quotient is 0.2 which is 20%.

Assessment
Student self-assessment page 151 Have students
circle one of the two choices to describe how they
feel about this activity.

Assessment tips Do students use mental math
skills to find a percent of a number and add or
subtract it from the original amount? Can students
mentally compute the percent change between
two numbers?
Materials
Student page 152
Protractors
Overhead projector and transparency with angles (optional)
Chart paper and markers

Concepts and Handbook References
Measure angles. (MOC 512)
Identify characteristics of plane figures. (MOC 339, 343)

Get Started
If you have an overhead projector, draw an angle on a transparency and ask for a student volunteer to demonstrate how to measure an angle using a protractor. Otherwise, ask them to follow along as you describe angle-measurement.

- Place the vertex of the angle at the center point of the flat side of the protractor.
- Be sure that one side of the angle goes through a zero degree mark.
- Read the angle measure on the same scale as the zero degree mark you used.

Today's Challenge
Student page 152 Tell students to use their angle-measuring skills to find some data about exterior angles in a polygon.

As students work at measuring the angles, circulate and help students with these hints.
- Be sure to measure only the numbered exterior angles.
- Turn the figure until you can comfortably read the measure.
- Be sure to use the protractor correctly.

Answers for student page 152: Expect slight variation in these measures. 1. $\angle 1: 150^\circ$; $\angle 2: 70^\circ$
$\angle 3: 140^\circ$ 2. $\angle 1: 100^\circ$; $\angle 2: 80^\circ$; $\angle 3: 80^\circ$; $\angle 4: 100^\circ$
3. $\angle 1: 90^\circ$; $\angle 2: 68^\circ$; $\angle 3: 72^\circ$; $\angle 4: 60^\circ$; $\angle 5: 70^\circ$
4. $\angle 1: 65^\circ$; $\angle 2: 60^\circ$; $\angle 3: 70^\circ$; $\angle 4: 50^\circ$; $\angle 5: 55^\circ$; $\angle 6: 60^\circ$

Look Ahead
On chart paper, prepare a copy of the table on student page 152.
**Materials**
Student page 153  
Protractors and rulers  
Graph Paper  

**Concepts and Handbook References**  
Measure angles. (MOC 330, 512)  
Identify characteristics of plane figures. (MOC 339, 343)  

**Get Started**
Have students post their angle-sums in the chart you prepared last time. In most cases, students will not have sums of exactly 360°. Discuss why their measures might vary, remeasure, and then settle on an average number for each.  

**Go Further**
**Student page 153** Challenge students to think about how the angle-sums could be 360° since polygons with many sides have many exterior angles. Provide graph paper for students who wish to use it for exercise 1.  

**Answers for student page 153:** 1. Check students’ work  
2. 360°  
3. 120°  
4. 45°  
5. Although a quadrilateral has more exterior angles than a triangle, its angles have smaller measures. Sample explanation:  

3. rotate and slide to C  
4. rotate and slide to D  
5. rotate and slide to A  
1. align a pencil on extension of one side at A  
2. rotate the pencil and slide to B  

**Assessment**
**Student self-assessment page 153** Have students circle one of the two choices to describe how they feel about this activity.  

**Assessment tip** Are students able to accurately measure angles with the protractor?
**Materials**
Student page 154
Calculators
Slips of paper containing three-digit numbers

**Concept and Handbook Reference**
Use divisibility rules. (MOC 069)

**Background**
For a review of divisibility rules, see page 70.

**Get Started**
Ask students to use the divisibility rules and name at least two numbers that divide evenly into 336 (2, 3, 4, 6), 700 (2, 4, 5, 10), and 824 (2, 4). For numbers for which they have no rules (as 11), students may need to divide.

**Today's Challenge**
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with one of these numbers written on it: 324, 326, 327, 328, 335, 339, 340, 342, 348, 351, 352, 354, 356, 370, 378. You do not need to use all of the numbers, but be sure that one student receives the number 326, since that number will be the “Fantastic Finalist.”

Have all students hold their numbers and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
- If your number is divisible by 10, sit down. (340, 370)
- If your number is divisible by four, sit down. (324, 328, 348, 352, 356)
- If your number is divisible by five, sit down. (335)
- If your number is divisible by six, sit down. (342, 354, 378)
- If your number is divisible by three, sit down. (327, 339, 351)

At this point, only the student holding the number 326 should still be standing. That student is the “Fantastic Finalist.”

**Go Further**
Student page 154 Have students complete the activity on the student page.

**Answers for student page 154:**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>416</td>
<td>845</td>
<td>608</td>
<td>750</td>
<td></td>
</tr>
<tr>
<td>132</td>
<td>209</td>
<td>264</td>
<td>534</td>
<td></td>
</tr>
<tr>
<td>545</td>
<td>600</td>
<td>223</td>
<td>736</td>
<td></td>
</tr>
<tr>
<td>450</td>
<td>904</td>
<td>963</td>
<td>660</td>
<td></td>
</tr>
</tbody>
</table>

6. 223 7. no; it is not even 8. no; it does not divide evenly by two and nine 9. odd number; prime number

**Assessment**
Student self-assessment page 154 Have students circle one of the two choices to describe how they feel about this activity.

**Assessment tips** Can students apply the divisibility rules to numbers greater than 100?
Materials
Student page 155

Concepts and Handbook References
Evaluate algebraic expressions. (MOC 206)
Examine functions and explain how dependent and independent variables relate. (MOC 234, 236)
Write a clear, complete constructed response. (MOC 531)

Background
Algebraic functions are relations in which a given value of one variable results in a unique value of a second variable.

Examples:
• \( x = y^2 \) is not a function because there are two possible values of \( y \) for a given value of \( x \).
  \( (4 = 2^2 \) and \( 4 = (-2)^2) \)
• \( y = 2x + 6 \) is a function because for every value of \( x \), there is one value for \( y \).

Get Started
Be sure students understand that when a function produces sets of ordered pairs \((x, y)\), the variable whose value is driven by the value of the independent variable is called the dependent variable. This means that as soon as you choose a value for \( x \), the value for \( y \) is established because \( y \) depends on \( x \).

Try this example with the class.
\( y = \frac{3 + x}{5 + x} \)
• When \( x \) is positive, what happens to the value of \( y \)? (It is a fraction whose value is always less than one.)
• When \( x \) is negative, what happens to the value of \( y \)? (If \( x \) is between zero and \(-3\), then \( y \) is a fraction between zero and one; if \( x = -3 \), then \( y = 0 \); if \( x \) is between \(-3 \) and \(-5 \), then \( y \) is negative; if \( x = -5 \), then \( y \) is undefined; if \( x \) is less than \(-5 \), then \( y \) is greater than one.)

Student page 155 Give students a few minutes to study the example of a constructed response at the top of student page 155. Discuss the example, paying particular attention to the variety of number types chosen to be sure the range of values for \( x \) has been covered.

Today’s Challenge
Student page 155 You may wish to let students work in pairs on Today’s Challenge. Circulate as they work and be sure they are testing values for \( w \) that will give them a clear picture of what can happen to \( s \).

Answers for student page 155: 1. Possible answer:
Try \( w = 0 \).
\( s = 7 - \frac{5}{3} = \frac{16}{3} \)
Try values of \( w \) between zero and one.
If \( w = 0.5 \)
\( s = 7 - \frac{2.5}{3} = \frac{16}{3} \)
If \( w = 0.7 \)
\( s = 7 - \frac{2.1}{3} = \frac{16}{3} \)
Try whole-number values of \( w \).
If \( w = 1 \)
\( s = 7 - \frac{1}{3} = \frac{16}{3} \)
If \( w = 10 \)
\( s = 7 - \frac{10}{3} = \frac{16}{3} \)
If \( w = 40 \)
\( s = 7 - \frac{40}{3} = \frac{16}{3} \)

Compare the values of \( s \).
\[
\begin{array}{c|c}
 w & s \\
\hline
 0 & 7 \\
 0.5 & \frac{16}{3} \\
 0.7 & \frac{16}{3} \\
 1 & \frac{16}{3} \\
 10 & \frac{16}{3} \\
 40 & \frac{16}{3} \\
\end{array}
\]

Write a clear, complete answer to the question.
In \( s = 7 - \frac{5}{3} \), as \( w \) gets larger, \( s \) gets smaller. Since there is no limit on how large \( w \) can be, there must also not be a limit on how small \( s \) can be. Since \( w \) must be greater than or equal to zero, the maximum value of \( s \) is 7 for \( w = 0 \).

2. Check students’ advice. They should mention that a variety of number types must be inspected.

Go Further
On a separate sheet of paper, have each student write his or her name and create a problem similar to those on student page 155. They should write the main points of a good constructed response on the back. Have them share their problem with a friend, who will solve it, then sign his or her name. If there is a disagreement, students should edit their work.

Assessment
Student self-assessment page 155 Have students circle one of the two choices to describe how they feel about the activity.

Assessment tip Can students substitute values into an algebraic expression and evaluate it?
Materials
Student page 156
Math Maze Cards (Week 32 Activity 156)

Concept and Handbook Reference
Compare two quantities. (MOC 049)

Background
When comparing two quantities, the number further to the right on the number line will be greater. If the quantities are both negative, help students visualize the location of the numbers before comparing. Students compare two positive numbers (4 > 3) more easily than two negative numbers (−4 < −3).

Get Started
Review the comparison symbols, <, >, and =, then practice a few comparisons. Ask students to explain their answers. Locating the quantities on a number line might be helpful.

• Which is greater, $\frac{2}{3}$ or 0.75? (0.75; 0.75 = $\frac{3}{4}$ and $\frac{2}{3} = \frac{2}{3}$)

• Which is greater, $-5 \times 6$ or $-100 + 72$?
  ($-100 + 72 = -28$ and $-28 > -30$)

• Which is greater, 0.901 or 0.910? (0.910; think about the fraction equivalents to these decimal numbers: the denominators are the same, so compare the numerators: $910 > 901$)

• Which is greater, $(-10)^2$ or $50 \times (-20)$? (They have the same value; $(-10)^2 = -1000$ and $50 \times (-20) = -1000$

Today's Challenge
Distribute the 18 Math Maze cards for week 32. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for this Math Maze.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 208.

Student page 156 When the group has finished playing the game, have students open their books and complete the Today's Challenge activity on page 156 in the student book.

Answers for student page 156: 1. $\frac{5}{12}$ of 150; 1
  2. $3 \times 10^3$; 2000 3. $\frac{7}{3} \times \frac{1}{2}$ 4. $(-20)^2 \times 5$; 1200
  5. $\frac{35}{400} = \frac{3}{40}$ 6. $30 = -8 \times 7$; 16 7. $68 = (-34); 2$
  8. $25\%$ of $(-40)$; 20 9. $0.7$; $0.63$ 10. $80 \times (-6)$;

Go Further
Student page 156 Have students share their own comparison puzzles with a friend.

Answers for student page 156: 21. Check students' work.

Assessment
Student self-assessment page 156 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students read decimal numbers confidently? Do they compare negative numbers correctly?
Materials
Student page 157
Calculators
Centimeter rulers
Chart paper and markers

Concepts and Handbook References
Use the Look for a Pattern and Draw a Diagram problem-solving strategies. (MOC 484, 483)
Use ratios. (MOC 424-426)
Identify characteristics of plane figures. (MOC 363)
Apply the Triangle Inequality Theorem and the Pythagorean Theorem. (MOC 350, 359)

Get Started
Draw a large square on the board. Ask students to imagine the diagonal for your square. Is the diagonal the same length as a side? Is it longer? Is it shorter? Have students draw a square and measure sides and diagonals to be sure that the diagonal is always longer. Relate this discovery to what they know about triangles: the sum of the lengths of any two sides is always greater than the length of the remaining side (Triangle Inequality Theorem) and the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse (Pythagorean Theorem).

Today's Challenge
Student page 157 Tell students to be sure to make accurate drawings and careful measurements. Ask them to sit in small groups so they can have access to calculators, but encourage independent work. Circulate and observe students as they work.
• Students should have drawn accurate squares, not rhombuses.
• Students should be measuring diagonals to the nearest millimeter (tenth of a centimeter).

Answers for student page 157: Assume that there will be slight measurement errors.

<table>
<thead>
<tr>
<th>Side length</th>
<th>Diagonal Length (to the nearest millimeter)</th>
<th>Diagonal Length (to the nearest tenth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 millimeters</td>
<td>14 millimeters</td>
<td>1.4</td>
</tr>
<tr>
<td>25 millimeters</td>
<td>35 millimeters</td>
<td>1.4</td>
</tr>
<tr>
<td>30 millimeters</td>
<td>42 millimeters</td>
<td>1.4</td>
</tr>
<tr>
<td>40 millimeters</td>
<td>57 millimeters</td>
<td>1.4</td>
</tr>
<tr>
<td>50 millimeters</td>
<td>71 millimeters</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Look Ahead
Prepare a chart like the table on student page 157 to fill in next time. Stress the importance of accurate data for next time when students will use their class data to make a conjecture about squares and diagonals.
Data 2 Analyze Patterns

Materials
Student page 158
Calculators

Concepts and Handbook References
Use the Look for a Pattern and Draw a Diagram problem-solving strategies. (MOC 484, 483)
Use ratios. (MOC 424–426)
Identify characteristics of plane figures. (MOC 363)
Write an equation to describe a functional relationship. (MOC 205, 244)

Get Started
With students' help, post data from student page 157. (See answers for student page 157.) If the students' results do not agree, discuss reasons this might be so (measuring error or measurement rounding, inaccurate diagrams, and so forth). Probe for agreement on the data so that the ratio element in the tables will be reasonably close to 1.4.

Go Further
Student page 158 Have students work in pairs or small groups with access to calculators. If students have trouble comparing their equations with the Pythagorean Theorem, ask them to think about values of \( a^2 + b^2 \) when the legs of the triangle are the sides of a square.

Answers for student page 158: Check students' work. Possible answers are provided.

Things we discovered:
• The ratio of side length to diagonal length in a square is about 1.4 no matter how long the sides are.
• If you multiply each side of a square by \( n \), then the length of the diagonal is also multiplied by \( n \).
• The diagonal is always longer than the side and the difference gets greater as the side gets longer.

1. about 98 mm 2. diagonal \( = 1.4 \times \text{side} \) 3. The Pythagorean Theorem states that \( a^2 + b^2 = c^2 \). If \( c \) is the diagonal of a square, it is also the hypotenuse of a right triangle in which \( a = b \), so you could say \( a^2 + a^2 = c^2 \) in this case. Solve for \( c \), the diagonal:

\[
\begin{align*}
\alpha^2 + \alpha^2 &= c^2 \\
2\alpha^2 &= c^2 \\
\sqrt{2\alpha} &= c \\
1.4\alpha &= c
\end{align*}
\]

Assessment
Student self-assessment page 158 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Did students state an accurate conjecture about the relationship between lengths of sides and diagonals in squares?
Materials
Student page 159
Calculators

Concept and Handbook Reference
Compute and compare unit costs. (MOC 438)

Background
The unit price or unit cost of an item is the cost per some unit of measure. To compute the unit cost for an item, divide the total price by the total number of units. Round to the nearest cent if needed.

Get Started
Write these two prices on the board.

12 ounces for $1.89 16 ounces for $2.25

Ask students to compute the cost per ounce for each item. (1.89 ÷ 12 = 0.1575 and 2.25 ÷ 16 = 0.140625) Explain that this is called the unit price for the item. The first item is approximately 16¢ per ounce and the second item is about 14¢ per ounce. The second item has the lower unit price and, if quality is the same, is a better buy. Repeat this for two additional items.

Student page 159 Have students complete the table in the Get Started section.

Answers for student page 159:

<table>
<thead>
<tr>
<th>Item 1</th>
<th>Unit Price</th>
<th>Item 2</th>
<th>Unit Price</th>
<th>Better Buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 oz for 69¢</td>
<td>8.6¢ per oz</td>
<td>10 oz for 85¢</td>
<td>8.5¢ per oz</td>
<td>about the same</td>
</tr>
<tr>
<td>2 pounds for $3.40</td>
<td>$1.70 per pound</td>
<td>3 pounds for $4.20</td>
<td>$1.40 per pound</td>
<td>Item 2</td>
</tr>
<tr>
<td>4 cans $2.25</td>
<td>$0.563 per can</td>
<td>6 cans for $3.50</td>
<td>$0.583 per can</td>
<td>Item 1</td>
</tr>
</tbody>
</table>

Today's Challenge
Explain that today you will be playing a game called "Who Wants to Be the Top Scorer?" Have each student take a blank sheet of paper and write an item's price and weight. Suggest that they think about items that they may have purchased at a store. Have the students calculate the unit price for their item to the nearest cent, then ask them to number their papers from 1 through 5.

As you ask each question, have students look at their items and unit prices, then answer the question. Yes answers will score points.

1. Does your item sell for more than $5.00? If yes, score 10 points.
2. Is the number of units in your item less than four? If yes, score 5 points.
3. Is your unit price a terminating decimal? If yes, score 8 points.
4. Is your unit price less than $0.05 or greater than $0.95 per unit? If yes, score 9 points.
5. Did you need to divide by a number greater than 25? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student write his or her problem on the board and explain how the points were scored. The maximum possible score is 42. If no student got that score, try to find it with the class. (Possible answer: $5.20 for 130 things = $0.04 per thing) If any student earned no points, have him or her explain the scoring, otherwise work together to create a zero-point score. (Possible answer: $4.33 for 6 ounces $0.72 per ounce)

Go Further
Student page 159 Have students solve the riddle, write and solve their own riddle, and create another riddle for a friend to solve. Have the solver sign his or her name.

Answers for student page 159: 2. about $0.32 3–4. Check students' riddles.

Assessment
Student self-assessment page 159 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students calculate unit prices and determine the best buy?
Materials
Student page 160
Chart or overhead transparency of blank fill-in grid

Concepts and Handbook References
Estimate and compute with decimal numbers.
(MOC 021, 176–178, 184–186)
Use a machine-scoring style fill-in grid. (MOC 532)

Background
Estimating division with decimals can take several forms.

Examples:
112.8 ÷ 0.24 → 113 × 4 Dividing by a number close to \( \frac{1}{4} \) is like multiplying by four.
7.3 ÷ 0.32 → 75 ÷ 3 Multiplying divisor and dividend by the same number does not change the quotient.

Get Started
Display the blank fill-in grid and demonstrate its use. Be sure students understand that the machine does not care whether an answer with fewer than four characters starts in the left-most column or ends in the right-most column.

Talk about how it may be more efficient to estimate and look for an answer choice that is close to the estimate than to spend a lot of time computing during a timed test. If you can do that, it is obvious that you understand the concept being tested.

Student page 160 Ask students to look at the top of student page 160. Ask a student to read exercise A aloud. Ask others to volunteer to tell how they would estimate the answer. (752 is about 750 and 1.3 is about 1.5, so the quotient is about the same as 7500 ÷ 15, or about 500) Repeat with exercise B. (112.8 is close to 112 and 0.24 is about \( \frac{1}{4} \), so the quotient is about 112 × 4, or 448.)

Today's Challenge
Student page 160 Be sure to remind students to explain their reasoning on the lines provided.


Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 160. Remind them to include matching answer choices. They should write the correct answers on the back. Have them share their problems with a friend, who will reason out (instead of computing) the correct answers, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 160 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students use estimation to eliminate incorrect answer choices?
**Materials**
Student page 161
Math Maze Cards (Week 33 Activity 161)

**Concepts and Handbook References**
Use order of operations, including exponents and integers. (MOC 207–209)
Order integers. (MOC 049)

**Background**
See page 41 for a review of order of operations.

**Get Started**
Review order of operations and evaluate, with the class, the following expressions.
- \(2 \times 3 - 6 \times 0.5\) (3)
- \(-100 - 7 \times -10\) (-30)
- \((-4)^2 \div 8\) (2)
- \(2 \times 5^2 + 6\) (56)

Review how to read and evaluate expressions with grouping symbols. Practice with these expressions.

<table>
<thead>
<tr>
<th>See</th>
<th>Say</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4^2)</td>
<td>the square of four</td>
<td>16</td>
</tr>
<tr>
<td>((-4)^2)</td>
<td>the square of negative four</td>
<td>16</td>
</tr>
<tr>
<td>(-4^2)</td>
<td>the opposite of the quantity four squared</td>
<td>-16</td>
</tr>
</tbody>
</table>

**Today’s Challenge**
Distribute the 18 Math Maze cards for week 33. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today’s Math Maze.

**Instructions for playing Math Maze** Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 209.

**Student page 161** When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 161 in the student book.

**Answers for student page 161:**
1. 75  2. -1  3. 81  4. -48  5. 5  6. -72  7. -10  8. 3  9. 15  10. -60  11. 2  12. 95  13. 55  14. 30  15. MATH POWER IS FUN

**Go Further**
**Student page 161** Have students complete this section of the student page.

**Answers for student page 161:**
16. Check students’ work.

**Assessment**
**Student self-assessment page 161** Have students circle one of the two choices to describe how they feel about this activity.

**Assessment tip** Do students remember to follow the order of operations when writing and evaluating expressions?
Materials
Student page 162
Calculators
Centimeter rulers

Concepts and Handbook References
Measure and compute with metric measures. (MOC 535)
Find ratios. (MOC 424-425)

Get Started
Draw an equilateral triangle and its altitude on the board. Ask students which segment is longer, the side or the altitude. Suggest that their lengths are related. If you know one length, you can figure out the other one.

Remind students that millimeters are tenths of centimeters, so for example 22 millimeters can also be written as 2.2 centimeters.

Today's Challenge
Student page 162 Pass out centimeter rulers and remind students to measure carefully. Have them sit in groups with access to calculators.

Answers for student page 162: 1. Students' measures should be close to these.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Height (to nearest millimeter)</th>
<th>Length of Side (to nearest millimeter)</th>
<th>Height/Length (to nearest tenth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2</td>
<td>2.5</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>4</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>5.2</td>
<td>6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

2. 6.1 cm; 0.9

Look Ahead
Tell students that next time they will use their data to find a new way to compute the area of an equilateral triangle.
Materials
Student page 163
Calculators
Centimeter rulers
Protractors

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Measure and compute with metric measures. (MOC 535)
Study relationships among segments in right triangles. (MOC 359)
Write an equation to describe a functional relationship. (MOC 205, 244)

Background
The Pythagorean theorem confirms the accuracy of the formula for finding the length of the altitude of an equilateral triangle.

Get Started
Have students post their results from activity 162. Remind them that slight measurement differences are to be expected.

Go Further
Student page 163 Have students work in pairs or small groups with access to calculators, centimeter rulers, and protractors. Remind them that the angles of an equilateral triangle all measure 60°. Once students understand that the ratio between the height and side length is consistent, challenge them to use what they have learned to compute the area of a triangle given only the side length.

Answers for student page 163: Check students' work. Possible answers are provided.

Things we discovered:
• The ratio of altitude to side length is very consistent.

If exercises 1–2 are incorrect but students have corrected them in exercise 3, give credit. 1. \( h = 0.9s \) where \( h \) is length of altitude and \( s \) is length of side
2. 4.5 cm 3. Check students' work. 4. 115.2 square centimeters; multiply 16 by 0.9 to get the height, then multiply that by half the length of a side.

Assessment
Student self-assessment page 163 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students compute the area of an equilateral triangle given only the side length?
Materials
Student page 164
Blank paper (heavyweight if possible) or index cards

Concept and Handbook References
Practice spatial visualization with reflections and rotations. (MOC 386, 388)

Background
There are three common transformations: reflection (flip), rotation (turn), and translation (slide). When an image is transformed using one or more of these, it remains congruent to the original.

Get Started
Review reflections and rotations with students. Hold an object such as a pencil, small flag, or heart-shaped paper. Ask students to visualize what a rotation of 90° or 180°, or a reflection about a vertical or horizontal line would look like. Then demonstrate by performing the transformation.

Student page 164 Have students complete the two examples at the top of student page 164.

Example 1:

Example 2:

Today's Challenge
Student page 164 Have students complete the table on student page 164. In each case, students are to sketch the image according to the reflection or rotation described.

Answers for student page 164:
1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 

Go Further
Have pairs of students make a set of cards to play the game “Concentration.” Each pair of students will need 12 small pieces of paper or 12 index cards. Have the students use one slip of paper or card to copy each sketch from exercises 1–10.

Instructions for playing “Concentration” Shuffle the cards and lay them facedown in a 5 × 2 array. The first player turns over any two cards. If the cards match (show congruent figures), the player keeps the cards and goes again. If the cards do not match, the player turns the cards back over and the other player takes a turn. Play continues until all cards have been taken. The player with more cards at the end of the game wins.

Assessment
Student self-assessment page 164 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students visualize and sketch the rotation or reflection of a figure?
Materials
Student page 165

Concepts and Handbook References
Make tally charts and histograms. (MOC 295)
Compare bar graphs and histograms.
(MOC 292, 295)
Write clear comparisons. (MOC 531)

Background
A histogram is a bar graph in which each bar represents an interval rather than a particular element. An axis in a histogram is set up so that each bar represents the interval whereas the corresponding axis in a bar graph is set up so that each bar represents an element such as a date, a place, or a quantity. Because the bars in a histogram represent intervals, there is no space between bars.

Get Started
Ask students to suggest some topics for which a bar graph would nicely display the data. If they need you to suggest a few topics to get the discussion going, you might propose a comparison of home runs hit by members of the school softball team or a survey of the kinds of shoes worn by classmates.

Explain that there is a type of graph that looks like a bar graph but allows for many data points by making each bar represent an interval, usually the same width.

Student page 165 Have students use the data about bird species sighted during the 2002 Great Backyard Bird Count to make a tally chart.

Answers for student page 165: 1.

<table>
<thead>
<tr>
<th>Number of Species</th>
<th>Number of States</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–74</td>
<td>\text{hit} II</td>
</tr>
<tr>
<td>75–149</td>
<td>\text{hit hit hit hit hit hit} \text{III}</td>
</tr>
<tr>
<td>150–224</td>
<td>\text{hit III}</td>
</tr>
<tr>
<td>225–299</td>
<td>\text{II}</td>
</tr>
</tbody>
</table>

Today's Challenge
Student page 165 Have students make a histogram for the tally chart they completed for the Get Started section. You may wish to let them work in pairs to complete exercises 4 and 5.

Answers for student page 165:

2. \text{Number of Species} \text{Number of States}
   \begin{tabular}{|c|c|}
   \hline
   0–39 & II \\
   40–79 & \text{hit II} \\
   80–119 & \text{hit hit III} \\
   120–159 & \text{hit hit II} \\
   160–199 & \text{III} \\
   200–239 & II \\
   240–279 & I \\
   \hline
   \end{tabular}

4. Check students’ work. The shape of the graph is much the same but the middle of the graph spreads out and does not show such a big spike in the second graph as in the first. 5. Check students’ work. Advice might include using smaller intervals.

Go Further
On a separate piece of paper have each student write his or her name and create a tally chart for an interval different from 40 or 75 for the Bird Count data. They should draw the associated histogram on the back. Have students share their tally charts with a friend, who will make the histogram and sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 165 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students comfortable with the differences between bar graphs and histograms?
Materials
Student page 166
Math Maze Cards (Week 34 Activity 166)

Concept and Handbook Reference
Recognize and extend a numerical pattern.
(MOC 484)

Get Started
Practice describing and extending numerical patterns with these examples.
• Describe the pattern and find the next number: -2, -4, -8, -16, ... (multiply n by two to get the next number; -32)
• Describe the pattern and find the next number: 2, 3, 5, 7, ... (prime numbers; 11)
• Describe the pattern and find the next number: 28, 26, 23, 19, ... (subtract consecutive integers starting with two; 14)
• Describe the pattern and find the next number: 4, 1, -2, -5, ... (subtract three; -8)

Today's Challenge
Distribute the 18 Math Maze cards for week 34. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today’s Math Maze.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 210.

Student page 166 When the group has finished playing the game, have students open their books and complete the Today’s Challenge activity on page 166 in the student book.

Answers for student page 166: 1. 13, 19, 22
2. -10, -40, -80 3. 5, 10, 15 4. 5, -1, -10 5. 9, 25, 36 6. 13, 18, 24 7. -7, -3, 1 8. -10, -15, -21

Go Further
Student page 166 Have students carefully write and edit sequence problems that work.

Answers for student page 166: 9. Check students’ work.

Assessment
Student self-assessment page 166 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students able to recognize a pattern in a numerical sequence and extend that pattern?
Materials
Student page 167
Calculators
Rulers

Concepts and Handbook References
Use the Look for a Pattern and Make a List problem-solving strategies. (MOC 484, 480)
Study the Fibonacci Sequence. (MOC 546)

Background
The Fibonacci Sequence is named for an Italian mathematician, Leonardo of Pisa, better known as Fibonacci. He was the first European mathematician to use the modern Hindu-Arabic numerals in his writings. He wrote his best known mathematics book in 1202 and in it included the rabbit problem on student page 167. The terms in the resulting sequence, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... are formed by finding the sum of the previous two terms. The ratio of any term to the preceding term approaches 1.618 as more and more terms are added to the sequence.

Get Started
Pose this problem.

A pair of rabbits produces a new pair each month. Each new pair produces a pair of rabbits after one month. How many pairs of rabbits will there be in four months?

Help students diagram a solution.

<table>
<thead>
<tr>
<th>Beginning of Month</th>
<th>Diagram</th>
<th>Pairs of Rabbits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P_1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>P_1, P_2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>P_1, P_2, P_3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>P_1, P_2, P_3, P_4, P_5</td>
<td>8</td>
</tr>
</tbody>
</table>

Today's Challenge
Student page 167 Remind students that the diagram or display they use will be critical to their finding the correct data.

Circulate and observe students as they work.
- Check to be sure students are finding the number of pairs of rabbits and not the number of rabbits.
- Be sure they see that this problem differs from the Get Started example in the number of months before a pair starts producing other pairs.
- You may need to help students to design an accurate way to portray the numbers of rabbit pairs per month.

Answers for student page 167: 1.

<table>
<thead>
<tr>
<th>Beginning of Month</th>
<th>Diagram</th>
<th>Pairs of Rabbits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P_1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>P_1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>P_1, P_2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>P_1, P_2, P_3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>P_1, P_2, P_3, P_4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>P_1, P_2, P_3, P_4, P_5</td>
<td>8</td>
</tr>
</tbody>
</table>

Look Ahead
Tell students that next time they will use the data to generate a pattern to help solve the original problem.
Materials
Student page 168

Concepts and Handbook References
Use the Look for a Pattern and Make a List problem-solving strategies. (MOC 484, 480)
Write an equation to describe a functional relationship. (MOC 205, 244)
Study the Fibonacci Sequence. (MOC 546)

Get Started
Have student volunteers post their rabbit-problem diagrams. If they disagree, have students discuss the problem until they reach consensus. (At the beginning of month six, there are eight pairs.)

Go Further
Student page 168 Have students work in pairs on today's activity.

Answers for student page 168: Check students’ work. Possible answers are provided.
Things we discovered:
• The number of pairs does not change by the same amount every month.
• The number pairs in a month is the sum of the number of pairs in the previous two months.

1. To find the number of pairs in month $n$, $P_n$, add the number of pairs in the preceding two months, $P_{n-1} + P_{n-2}$. 2. 144 3. 610

Assessment
Student self-assessment page 168 Have students circle one of the two choices to describe how they feel about this activity.
Assessment tips Did students effectively portray the rabbit problem to compute the number of rabbit pairs. Were they able to find the pattern to the rabbit problem?
Materials
Student page 169
Math Jumble activity poster and shape cards

Concept and Handbook References
Review attributes of polygons. (MOC 351–352, 358, 363, 389)

Background
See page 89 for a review of naming schemes for polygons.

Get Started
To review the names and properties of common polygons, draw these figures on the board. Be sure to include the labels in your drawings.

Figure 1
Figure 2
Figure 3
Figure 4

Discuss attributes of each figure.
- Figure 1: Right isosceles triangle (one right angle, two congruent angles, two congruent sides, one line of symmetry)
- Figure 2: Isosceles trapezoid (a pair of parallel sides, a pair of congruent sides (legs), one line of symmetry, two pairs of congruent base angles.)
- Figure 3: Regular hexagon (all sides congruent, all angles congruent, three pairs of parallel sides, six lines of symmetry)
- Figure 4: Parallelogram (two pairs of congruent and parallel sides, opposite angles congruent, adjacent angles supplementary (sum is 180°), no line of symmetry)

Today’s Challenge
Use the shape cards to construct this poster.

Explain that the object of today’s Math Jumble is to find three figures in a row, horizontally, vertically, or diagonally, that share a common attribute. Identify the three figures and name the common attribute.

Students may use any three in a row. For instance, the first three in the top row are all triangles. The first three in the right column are all regular polygons.

Student page 169 Have students use the Math Jumble on student page 169 to find three adjacent polygons that share a common attribute.

Answers for student page 169: 1–4. Check students’ grids, answers, and explanations of attributes. Possible answers are provided. G, K, M all have vertical symmetry; I, N, O are all scalene; G, H, K all have only one line of symmetry; A, B, F are all regular polygons.

Go Further
Student page 169 Have students draw one more figures that shares the common attributes found for exercises 1–4.

Answers for student page 169: Check students’ work.

Assessment
Student self-assessment page 169 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students identify a common attribute shared by polygons?
Materials
Student page 170

Concepts and Handbook References
Use similarity to solve problems. (MOC 376)
Use proportions. (MOC 428–429)
Use the *Draw a Diagram* problem-solving strategy. (MOC 483)

Background
In *similar figures*, the corresponding angles are congruent and corresponding sides are proportional. It is very important to establish corresponding sides before using proportions to solve for missing side-lengths.

Get Started
Tell students that sometimes you have to draw your own diagram to solve a problem. In that case it is important to make an accurate diagram before trying to compute the answer. Sometimes they will need to use margins, scrap paper, or the back of a test, but diagrams will help them to be sure they’re solving the correct problem!

Read this problem aloud. Ask students to make a diagram before they try to solve it.

Rectangle *QUIZ* is similar to rectangle *RANT*. The side lengths of *QUIZ* are 6 inches and 10 inches. The longest side of *RANT* is 30 inches long. How long is the shorter side of *RANT*?

Ask a volunteer to draw his or her diagram on the board, then check it against the original problem before working with the class to solve it. Look for reasoning along these lines.

- Similar figures have corresponding sides in the same ratio.
- A proportion may be solved using equivalent fractions or by finding cross-products.

\[
\frac{\text{short RANT}}{\text{short QUIZ}} = \frac{\text{long RANT}}{\text{long QUIZ}} \quad \frac{x}{6} = \frac{30}{10}
\]

OR

\[
\frac{\text{short RANT}}{\text{long RANT}} = \frac{\text{short QUIZ}}{\text{long QUIZ}} \quad \frac{x}{30} = \frac{6}{10}
\]

Either way, \(x = 18\); the short side of *RANT* is 18 inches long.

Student page 170 Work with the class on the exercise at the top of student page 170. Ask students to first draw the diagram before attempting to compute the answer.

(A. \(\overline{BE}\) B. \(\overline{WE}\) C. 12 ft)

Today's Challenge
Student page 170 Tell students that they will have to make careful drawings in order to solve these problems.

Answers for student page 170: Check students’ diagrams. 1. 15 ft 2. 9 cm 3. 20 ft 4. 15 in. 5. Check students’ advice. They should mention being careful about corresponding sides.

Go Further
On a separate sheet of paper, have each student write his or her name and create three problems similar to those on student page 170. They should write the correct answers on the back. Have them share their problems with a friend, who will draw a diagram, solve the problem, and then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 170 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students draw careful diagrams to help them solve for side lengths in similar figures?
Materials
Student page 171
Math Maze Cards (Week 35 Activity 171)

Concept and Handbook Reference
Relate side-length or radius, perimeter or circumference, and area of plane figures. (MOC 560–562)

Background

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter or Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>$4s$</td>
<td>$s^2$</td>
</tr>
<tr>
<td>Rectangle</td>
<td>$2l + 2w$</td>
<td>$lw$</td>
</tr>
<tr>
<td>Circle</td>
<td>$2\pi r$</td>
<td>$\pi r^2$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$s_1 + s_2 + s_3$</td>
<td>$\frac{1}{2}bh$</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>$2l + 2w$</td>
<td>$bh$</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>$s_1 + s_2 + s_3 + s_4$</td>
<td>$\frac{1}{2}h(b_1 + b_2)$</td>
</tr>
</tbody>
</table>

Get Started
Review perimeter and area. Generate a list of formulas for perimeter and area, along with a sketch of the polygon with the dimensions labeled. Keep it where students can see it and add to it.

Remind students that the diameter of a circle is twice the length of the radius. They should leave answers in terms of $\pi$ instead of multiplying by 3.14.

Today's Challenge
Distribute the 18 Math Maze cards for week 35. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game.

Instructions for playing Math Maze
Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, “Who has the card with the answer to the question just read?” Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student’s card.

The correct sequence of questions and answers is shown on page 211.

Student page 171
When the group has finished playing the game, have students open their books and complete the Today's Challenge activity on page 171 in the student book.

Answers for student page 171: 1. $P = 42$ cm; $A = 108$ sq. cm 2. $C = 40\pi$ cm; $A = 400\pi$ sq. cm 3. $P = 18$ cm; $A = 12$ sq. cm 4. $P = 44$ cm; $A = 121$ sq. cm 5. $P = 54$ cm; $A = 102$ sq. cm 6. $P = 52$ cm; $A = 128$ sq. cm

Go Further
Student page 171
Help students relate perimeter, area, and whole-number dimensions.

Answers for student page 171: 7. in any order: 1 cm and 36 cm; 2 cm and 18 cm; 3 cm and 12 cm; 4 cm and 9 cm; 6 cm and 6 cm 8. There are five sets of factors for 36 and each provides a possible whole-number base and height for a rhombus with area 36 square cm.

Assessment
Student self-assessment page 171
Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip
Can students reliably extrapolate missing information from given information?
Materials
Student page 172
Colored squares
Centimeter rulers

Concept and Handbook References
Write equations to describe functional relationships. (MOC 205, 244)

Get Started
Draw a two unit by three unit rectangle on the board. Ask these questions.
• How many unit squares are needed to cover the rectangle? (6)
• What is the perimeter of the rectangle? (10 units)
• How many unit squares are needed to surround the rectangle? (14)

The last result might surprise students. Suggest that there is a way to determine the number of unit squares needed to surround any rectangle but they will have to gather data to discover it.

Today’s Challenge
Student page 172 Stress the need to carefully draw rectangles. Have students sit in small groups with access to colored squares and rulers.

Answers for student page 172: 1–6. Check students’ work. Numbers in the last column are four greater than numbers in the middle column.

Look Ahead
Tell students that next time they will develop a formula for finding the number of unit squares needed to surround any rectangle.
Data 2

Materials
Student page 173

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Write an equation to describe a functional relationship. (MOC 205, 244)

Get Started
Have students post all their data from activity 172 on the board. Students will have used a number of different rectangular dimensions. Posting all of them will give students ample data for their patterns-search.

Go Further
Student page 173 Have students work in pairs. Challenge them to find a pattern that will help them tile a pool of any rectangular dimensions.

Answers for student page 173: Check students’ work. Possible answers are provided.

Things we discovered:
• The dimensions of a rectangle created by surrounding a rectangle with unit squares are two units longer than the original.
• The perimeter of the new rectangle is eight units greater than the perimeter of the original.

• The number of unit squares needed to surround a rectangle is not the same as the number of units in its perimeter because the corner squares are not on the perimeter.
• The number of unit squares needed to surround a rectangle is four more than the number of units in the perimeter—this accounts for the corner squares.

1. 144 2. Number of tiles = perimeter + four corner tiles \( t = p + 4 \) or \( t = 2l + 2w + 4 \) 3. No, 240 tiles could be used to surround pools with these dimensions: 117 \( \times \) 1, 116 \( \times \) 2, 115 \( \times \) 3, 114 \( \times \) 4, and so forth.

Assessment
Student self-assessment page 173 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students draw accurate and useful diagrams? Can they express a functional relationship with an equation?
Materials
Student page 174
Slips of paper containing decimals

Concept and Handbook Reference
Round decimal numbers to a given place.
(MOC 021)

Get Started
Review the rules for rounding a number to a specified place value. If the digit to the right of the round-to place is five or greater, add one to the round-to digit and drop all digits to its right. If the digit to the right of the round-to place is less than five, just drop it and all digits to its right. Practice with these problems.
• Round 0.483 to the nearest tenth (0.5) and nearest hundredth (0.48).
• Round 0.789 to the nearest tenth (0.8) and nearest hundredth (0.79).
• Round 0.796 to the nearest tenth (0.8) and nearest hundredth (0.80).

Today's Challenge
Explain that today the class will be playing a game called “Fantastic Finalist.” Give each student a piece of paper with one of these numbers written on it: 0.32, 0.38, 0.421, 0.449, 0.581, 0.591, 0.604, 0.63, 0.67, 0.69, 0.708, 0.738, 0.744, 0.78, 0.799, 0.8.

You do not need to use all of the numbers, but be sure that one student receives the number 0.32, since that number will be the “Fantastic Finalist.”

Have all students hold their numbers and stand in a large circle. Explain that the object of the game is to be the “Fantastic Finalist,” the last student to remain standing.

Read each of the following challenges.
• If your number rounded to the nearest tenth is 0.6, sit down. (0.581, 0.591, 0.604, 0.63)
• If your number rounded to the nearest tenth is 0.4, sit down. (0.38, 0.421, 0.449)
• If your number rounded to the nearest hundredth is 0.74, sit down. (0.738, 0.744)
• If your number rounded to the nearest tenth is 0.7, sit down. (0.67, 0.69, 0.708)
• If your number rounded to the nearest whole number is one, sit down. (0.78, 0.799, 0.8)

At this point, only the student holding the number 0.32 should still be standing. That student is the “Fantastic Finalist.”

Go Further
Student page 174 Have students complete the activity on the student page.

Answers for student page 174:
1–5.  

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.58</td>
<td>0.38</td>
<td>0.342</td>
</tr>
<tr>
<td>0.61</td>
<td>0.58</td>
<td>0.373</td>
</tr>
<tr>
<td>0.49</td>
<td>0.33</td>
<td>0.337</td>
</tr>
<tr>
<td>0.59</td>
<td>0.55</td>
<td>0.592</td>
</tr>
</tbody>
</table>

6. 0.497 7. 0.5 8. 0.50 9. 0

Assessment
Student self-assessment page 174 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Can students round decimal values to the specified place value?
Materials
Student page 175

Concepts and Handbook References
Analyze errors. (MOC 533)
Use scientific notation. (MOC 016)

Background
To write a number in scientific notation, write it as the product of a power of 10 and a decimal greater than or equal to one and less than 10.

Examples:
\[ 5.234 \rightarrow 5.234 \times 10^1 \]
\[ 0.006 \rightarrow 6 \times 10^{-3} \]

Get Started
Display this expression on the board: \( 3.4 \times 10^4 \). Ask students to explain how to write it as a whole number (3,400,000). Ask students why one would write a number in scientific notation. As part of the discussion, clarify these concepts.

• Scientific notation allows extremely large and small numbers to be written in a compact notation. Scientific notation also allows a quick comparison between extremely large numbers and between extremely small numbers.

• To multiply by a positive power of 10, shift the decimal point to the right the same number of places as indicated by the exponent.

Student page 175 Have students study the four equations at the top of student page 175. Let them work alone for a few minutes, then ask for a volunteer to explain why one example is correct or incorrect. If students in the class disagree, have the class continue to discuss the equation until they reach a consensus. Continue until all four examples have been analyzed. (A is correct; Greta seems to annex zeros instead of moving the decimal point as indicated by the exponent. B. 7100 C. 62,000 D. 445,000)

Today’s Challenge
Student page 175 Ask students to select problems they think Greta would evaluate correctly and those she would evaluate incorrectly, and to explain their thinking. Then, ask them to give some advice to Greta about scientific notation.

Answers for student page 175: 1. Greta would do D correctly. A. 5200 B. 68,000 C. 760 D. 8,000,000

Go Further
On a separate piece of paper have each student write his or her name and create three problems involving evaluating expressions written in scientific notation. One problem should be one that Greta would evaluate correctly, and two should be ones that Greta would not evaluate correctly. Have students share their problems with a friend, who is to identify the one problem that Greta would evaluate correctly, evaluate all three, and sign his or her name. If there is disagreement, students should edit their problems.

Assessment
Student self-assessment page 175 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students evaluate expressions represented in scientific notation?
Materials
Student page 176
Math Maze Cards (Week 36 Activity 176)
Boxes that can be unfolded

Concept and Handbook References
Find surface area and volume of rectangular prisms and cylinders. (MOC 401-402, 412-413)

Background
• The surface area of a figure is the total area of its faces. Surface area is measured in square units. Rectangular prisms have three congruent pairs of faces. A cylinder has two circular bases and the lateral surface, which is actually a rectangle whose base is the circumference of the circle.
• The volume of a three-dimensional figure is how much space it occupies. Volume is measured in cubic units. The following formulas will be helpful.
  \[ SA(\text{rectangular prism}) = 2lw + 2lh + 2wh \]
  \[ SA(\text{cylinder}) = 2\pi r^2 + 2\pi rh \]
  \[ V(\text{rectangular prism}) = Bh = \ell wh \]
  \[ V(\text{cylinder}) = Bh = \pi r^2h \]

Get Started
Students have difficulty remembering surface area and volume formulas. To help them think about surface area, unfold a donut-type box to show the faces (discounting the tabs and flaps). To show a cylinder, roll a sheet of paper into the cylindrical shape and then unfold it to show the lateral portion of the cylinder as a rectangle. To help students think about the volume of either figure, remind them that it is always the area of the base multiplied by the height.

Draw these two diagrams on the board. Ask students to find the surface area and volume of each.

- **Rectangular Prism**
  - SA = 90 square units
  - V = 54 cubic units

- **Cylinder**
  - SA = 54\pi square units
  - V = 54\pi cubic units

Today's Challenge
Distribute the 18 Math Maze cards for week 36. Each student should receive at least one card, but since all cards need to be distributed, some students may need to get more than one card. Use the cards to play the Math Maze game. Students may wish to have paper and pencil available for today's Math Maze.

Instructions for playing Math Maze Ask students to look at their cards. Ask one student to read the question that is written on his or her card. Next ask, "Who has the card with the answer to the question just read?" Ask that student to read the answer, and then read the question on his or her card. Play continues until all questions have been answered. The last answer to be read should be the answer on the first student's card.

The correct sequence of questions and answers is shown on page 212.

Student page 176 When the group has finished playing the game, have students open their books and complete the Today's Challenge activity on page 176 in the student book.

Answers for student page 176: 1. < 2. < 3. < 4. = 5. < 6. > 7. > 8. >

Go Further
Student page 176 Have students consider their answers to exercises 1–8 as they complete the remaining questions.

Answers for student page 176: 9. twice 10. half 11. yes; the top half will have the same volume as the bottom half, which is the original cylinder 12. no, not necessarily; explanations vary

Assessment
Student self-assessment page 176 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Are students able to correctly use the formulas for surface area and volume?
Materials
Student page 177
Rulers
Calculators

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Relate the Golden Ratio to the Fibonacci Sequence. (MOC 547, 546)
Compute with decimals. (MOC 186)
Find ratios. (MOC 424–425)
Compute with measures. (MOC 535)

Background
Some researchers have found that the most aesthetically pleasing rectangle is one whose sides are in the ratio of about 1.6 : 1. This rectangle is called the Golden Rectangle. The Golden Ratio is also the ratio between terms in the Fibonacci Sequence (See activities 167–168).

Get Started
Direct students to student page 177 and ask them to select the rectangle they find most aesthetically pleasing. Keep a tally to refer to next time.

Today's Challenge
Student page 177 Have students work in pairs or small groups with access to calculators. Encourage them to cooperate to complete this activity.

Answers for student page 177: Due to measurement error, answers will vary slightly. 1. A: 3.0; B: 4.0; C: 1.6; D: 1; E: 2.7; F: 1.3

2. Number Pair Ratio
1, 1 1 : 1
2, 1 2 : 1
3, 2 1.5 : 1
5, 3 1.7 : 1
8, 5 1.6 : 1
13, 8 1.6 : 1
21, 13 1.6 : 1
34, 21 1.6 : 1

Look Ahead
Tell students that next time they will use the data to find whether there are any Golden Rectangles in the classroom.
Materials
Student page 178
Rulers
Calculators

Concepts and Handbook References
Use the Look for a Pattern problem-solving strategy. (MOC 484)
Relate the Golden Ratio to the Fibonacci Sequence. (MOC 547, 546)

Get Started
Ask volunteers to read answers for activity 177. If there are disagreements, have them remeasure or recompute to be sure of their results. Ask which rectangle they think might be a Golden Rectangle. (C) Compare this response to the survey of pleasing rectangles you did last time.

Go Further
Student page 178 Tell students that the Golden Ratio is also found in many places in nature, from the spiral of an elephant’s trunk to the bones of their little finger. Have them work independently with access to rulers and calculators.

Answers for student page 178: Check students’ work. Possible answers are provided.

Things we discovered:
• The ratios in the Fibonacci numbers, after the first four, get to about 1.6, then stay about the same.
• Rectangle C has a ratio of 1.6 : 1.
1. About 1.6 : 1 2. Values will vary.

Assessment
Student self-assessment page 178 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tips Do students perceive the special qualities of the Golden Rectangle?
Materials
Student page 179
Straightedge (optional)

Concept and Handbook References
Identify line symmetry and rotational symmetry of quadrilaterals. (MOC 387, 389)

Background
See page 89 for a review of line symmetry.
A figure has rotational symmetry if you can rotate it 180° (or less) about a point and have the rotated figure exactly match the original figure.

Get Started
Draw these figures on the board.

One line of symmetry
Six lines of symmetry
Four lines of symmetry
No lines of symmetry
60° rotational symmetry
90° rotational symmetry
180° rotational symmetry

Discuss the lines of symmetry for each figure. Pass out graph paper and have students copy the figures and draw the lines of symmetry. Note that the regular hexagon has three lines of symmetry that pass through the midpoints of opposite sides and three that pass through opposite vertices.

Discuss the rotational symmetry for each figure. If a figure must be rotated more than 180° to match up with itself, it has no rotational symmetry.

Student page 179 Have students use the information on the board to answer the questions in the Get Started section.

Answers for student page 179:
1. 2. 180° 3. 4. 90°
5. 6. 90°
7. Both quadrilaterals have line symmetry through the midpoints of opposite sides. 8. Both quadrilaterals have line symmetry through opposite vertices; both have 90° rotational symmetry.

Today's Challenge
Explain that today you will be playing a game called “Who Wants to Be the Top Scorer?” Have each student take a blank sheet of paper and draw any polygon, then ask them to number their papers from 1 through 5.

As you ask each question, have students look at their polygons and answer the question. Yes answers will score points.

1. Does your polygon have exactly one line of symmetry? If yes, score 10 points.
2. Does your polygon have no line of symmetry? If yes, score 5 points.
3. Does your polygon have rotational symmetry of less than 180°? If yes, score 9 points.
4. Does your polygon have more than four lines of symmetry? If yes, score 8 points.
5. Is your polygon a regular polygon? If yes, score 15 points.

Have students find their total scores. Determine which student has the highest score. Have that student draw the polygon on the board and explain how he or she scored the points. If no student scored 32 points, work with the class to produce the maximum-point figure. (Any regular polygon with more than four sides will produce 32 points.)

Go Further
Student page 179 Have students solve the riddles and create another riddle for a friend to solve. Have the solver sign his or her name.


Assessment
Student self-assessment page 179 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students correctly identify line and rotational symmetry of quadrilaterals?
Materials
Student page 180
Calculators

Concepts and Handbook References
Find area. (MOC 356, 368)
Compute with measures. (MOC 357)
Use the Pythagorean Theorem. (MOC 359)
Write a clear, careful constructed response for a multi-step problem. (MOC 531)

Get Started
Review how to write a good constructed response. (Break down the problem into a series of steps; explain each step, showing computation and diagrams; use the wording of the original problem to help you write your answer.)

Student page 180 Ask students to work in pairs to study the Get Started problem and its solution. After a few minutes, discuss why this is a clear, complete constructed response. Ask students to suggest alternative ways to respond to the same question. Emphasize that, if they compute without keeping the units with the numbers, they must be very careful to use correct units in their answers.

Today's Challenge
Student page 180 Have students work in groups of two or three, sharing calculators. Students can consult with each other, but each should complete student page 180.

Answers for student page 180: Check students' work. Possible answers are provided.

1. Decide on the correct area formula.
   \( A = \frac{1}{2}bh \)
   \( b = \) length of base.
   \( h = \) length of altitude to base.
   The length of part of the base is missing.
   \( b_f + b_t = c \)
   \( 12 + b_t = 13 \)
   \( 144 + b_t^2 = 169 \)
   \( b_t^2 = 169 - 144 \)
   \( b_t = 5 \)

Now compute the area.
\( A = \frac{1}{2} \times (5 + 9) \times 12 \)
\( A = 84 \)

Write the answer to the question.
The area of the triangle is 84 square centimeters.

2. Decide on a useful area formula.
   \( A = \frac{1}{2}(b_1 + b_2)h \)
   \( b_1 \) and \( b_2 \) are lengths of the parallel sides.
   \( h \) is the altitude between \( b_1 \) and \( b_2 \).
   The length of the altitude is missing.

Because the figure is a trapezoid, the 10-foot and 18-foot sides are parallel. The two perpendicular segments connecting them make a rectangle, so \( MN \) is the same length as \( PQ \). This means that \( MN \) is eight feet long.
The altitude is a leg of a right triangle for which the length of a leg and the length of the hypotenuse are known.
\( a^2 + b^2 = c^2 \)
\( a^2 + 8^2 = 10^2 \)
\( a^2 = 100 - 64 \)
\( a^2 = 36 \)
\( a = 6 \)

Compute the area.
\( A = \frac{1}{2}(b_1 + b_2)h \)
\( A = \frac{1}{2}(10 + 18)6 \)
\( A = 84 \)
The area of the trapezoid is 84 square feet.

3. Check students' work.

Go Further
On a separate sheet of paper, have each student write his or her name and create one problem similar to those on student page 180. They should write the main points of a constructed response and the correct answer on the back. Have them share their problem with a friend, who will write a constructed response, then sign his or her name. If there is disagreement, students should edit their work.

Assessment
Student self-assessment page 180 Have students circle one of the two choices to describe how they feel about this activity.

Assessment tip Can students compute to find missing information as one step in a multi-step problem?
Questions and Answers for Math Maze Cards

The tables below show the sequence that the questions and answers should follow. Find the starting question. Look across the row for the correct answer. Then go on to the next question below and continue until you reach the end of the table. Then go to the top of the table and read down until you reach the starting question again.

<table>
<thead>
<tr>
<th>Who has $2^3 \times 3^2$?</th>
<th>I have 72.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has $5^2 \times 7$?</td>
<td>I have 175.</td>
</tr>
<tr>
<td>Who has $2 \times 5 \times 13$?</td>
<td>I have 130.</td>
</tr>
<tr>
<td>Who has $2^3 \times 3 \times 5$?</td>
<td>I have 60.</td>
</tr>
<tr>
<td>Who has $2^3 \times 5^2$?</td>
<td>I have 200.</td>
</tr>
<tr>
<td>Who has $2 \times 7^2$?</td>
<td>I have 98.</td>
</tr>
<tr>
<td>Who has $13^2$?</td>
<td>I have 169.</td>
</tr>
<tr>
<td>Who has $5^2$?</td>
<td>I have 125.</td>
</tr>
<tr>
<td>Who has $2 \times 3^3 \times 5$?</td>
<td>I have 270.</td>
</tr>
<tr>
<td>Who has $2 \times 5 \times 7^2$?</td>
<td>I have 490.</td>
</tr>
<tr>
<td>Who has $2 \times 5 \times 11$?</td>
<td>I have 110.</td>
</tr>
<tr>
<td>Who has $11^2$?</td>
<td>I have 121.</td>
</tr>
<tr>
<td>Who has $2^3 \times 5^2$?</td>
<td>I have 1000.</td>
</tr>
<tr>
<td>Who has $2^3 \times 5^2$?</td>
<td>I have 40.</td>
</tr>
<tr>
<td>Who has $3^2 \times 5^2$?</td>
<td>I have 225.</td>
</tr>
<tr>
<td>Who has $2^3 \times 7^2$?</td>
<td>I have 196.</td>
</tr>
<tr>
<td>Who has $3^2 \times 7^2$?</td>
<td>I have 63.</td>
</tr>
<tr>
<td>Who has $2 \times 3 \times 5 \times 7$?</td>
<td>I have 210.</td>
</tr>
</tbody>
</table>
## Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th><strong>Who has</strong></th>
<th><strong>I have</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the name of the set of numbers ({1, 2, 3, \ldots})?</td>
<td>counting numbers.</td>
</tr>
<tr>
<td>Who has the name of the addition property that states (4 + 7 = 7 + 4)?</td>
<td>commutative property.</td>
</tr>
<tr>
<td>Who has the name of a relationship between the numbers 4 and (\frac{1}{2})?</td>
<td>reciprocal.</td>
</tr>
<tr>
<td>Who has the name of a letter in an equation that can be replaced by a number?</td>
<td>variable.</td>
</tr>
<tr>
<td>Who has the name of the part of a fraction that tells the number of equal parts in one whole?</td>
<td>denominator.</td>
</tr>
<tr>
<td>Who has the name of the number that you divide by in division?</td>
<td>divisor.</td>
</tr>
<tr>
<td>Who has the name of a number like (\frac{2}{3}) or (-\frac{1}{3})?</td>
<td>rational number.</td>
</tr>
<tr>
<td>Who has the name of the property that states (3(8 + 2) = 3 \times 8 + 3 \times 2)?</td>
<td>Distributive Property.</td>
</tr>
<tr>
<td>Who has the name of the greatest number that divides evenly into two numbers?</td>
<td>greatest common factor.</td>
</tr>
<tr>
<td>Who has the name for the value of the variable that makes an equation true?</td>
<td>solution.</td>
</tr>
<tr>
<td>Who has the name of the set of whole numbers and their opposites?</td>
<td>integers.</td>
</tr>
<tr>
<td>Who has the name of the least number that two numbers divide into evenly?</td>
<td>least common multiple.</td>
</tr>
<tr>
<td>Who has the word that tells that two numbers or expressions have the same value?</td>
<td>equivalent.</td>
</tr>
<tr>
<td>Who has the name of a number that is part whole number and part fraction, like (1\frac{1}{2}) or (3\frac{3}{4})?</td>
<td>mixed number.</td>
</tr>
<tr>
<td>Who has the name of a number that has more than two factors?</td>
<td>composite number.</td>
</tr>
<tr>
<td>Who has the name of the addition property that states (2 + (8 + 4) = (2 + 8) + 4)?</td>
<td>Associative Property.</td>
</tr>
<tr>
<td>Who has the name for the number of times a base is used as a factor?</td>
<td>exponent.</td>
</tr>
<tr>
<td>Who has the name of a relationship between the numbers 4 and (-4)?</td>
<td>opposites.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the number of ounces in 1.5 pounds?</td>
<td>I have 24.</td>
</tr>
<tr>
<td>Who has the number of feet in a mile?</td>
<td>I have 5280.</td>
</tr>
<tr>
<td>Who has the number of milliliters in 0.25 liter?</td>
<td>I have 250.</td>
</tr>
<tr>
<td>Who has the number of pounds in $\frac{1}{4}$ ton?</td>
<td>I have 1500.</td>
</tr>
<tr>
<td>Who has the number of centimeters in 25 meters?</td>
<td>I have 2500.</td>
</tr>
<tr>
<td>Who has the number of cups in 3 quarts?</td>
<td>I have 12.</td>
</tr>
<tr>
<td>Who has the number of minutes in 2.5 hours?</td>
<td>I have 150.</td>
</tr>
<tr>
<td>Who has the number of millimeters in 2.5 centimeters?</td>
<td>I have 25.</td>
</tr>
<tr>
<td>Who has the number of months in $1\frac{1}{4}$ years?</td>
<td>I have 21.</td>
</tr>
<tr>
<td>Who has the temperature, in degrees Fahrenheit, at which water freezes?</td>
<td>I have 32.</td>
</tr>
<tr>
<td>Who has the number of feet in $5\frac{1}{2}$ yards?</td>
<td>I have 16.</td>
</tr>
<tr>
<td>Who has the number of inches in $\frac{1}{4}$ yard?</td>
<td>I have 27.</td>
</tr>
<tr>
<td>Who has the number of quarts in 6.5 gallons?</td>
<td>I have 26.</td>
</tr>
<tr>
<td>Who has the temperature, in degrees Celsius, at which water freezes?</td>
<td>I have 0.</td>
</tr>
<tr>
<td>Who has the number of grams in 25 kilograms?</td>
<td>I have 25,000.</td>
</tr>
<tr>
<td>Who has the number of pencils in 2.5 dozen pencils?</td>
<td>I have 30.</td>
</tr>
<tr>
<td>Who has the number of seconds in $\frac{5}{8}$ minute?</td>
<td>I have 50.</td>
</tr>
<tr>
<td>Who has the number of weeks in one year?</td>
<td>I have 52.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th><strong>Who has</strong> $n$ <strong>if</strong> $6n = 84$?</th>
<th><strong>I have</strong> 14.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{n}{4} = 13$?</td>
<td><strong>I have</strong> 52.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $n - 23 = 15$?</td>
<td><strong>I have</strong> 38.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $n + 13 = 38$?</td>
<td><strong>I have</strong> 25.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $7n = 84$?</td>
<td><strong>I have</strong> 12.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $15n = 60$?</td>
<td><strong>I have</strong> 4.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{1}{3}n = 15$?</td>
<td><strong>I have</strong> 45.</td>
</tr>
<tr>
<td><strong>Who has one value for</strong> $n$ <strong>if</strong> $n^2 = 121$?</td>
<td><strong>I have</strong> 11.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{n}{16} = 1$?</td>
<td><strong>I have</strong> 16.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{n}{3} = 9$?</td>
<td><strong>I have</strong> 27.</td>
</tr>
<tr>
<td><strong>Who has one value for</strong> $n$ <strong>if</strong> $n^2 = 169$?</td>
<td><strong>I have</strong> 13.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{1}{2}n = 18$?</td>
<td><strong>I have</strong> 54.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $7n = 0$?</td>
<td><strong>I have</strong> 0.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $80 - n = 16$?</td>
<td><strong>I have</strong> 64.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $4n = 96$?</td>
<td><strong>I have</strong> 24.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $n + 27 = 44$?</td>
<td><strong>I have</strong> 17.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{3}{2}n = 24$?</td>
<td><strong>I have</strong> 36.</td>
</tr>
<tr>
<td><strong>Who has</strong> $n$ <strong>if</strong> $\frac{n}{12} = 5$?</td>
<td><strong>I have</strong> 60.</td>
</tr>
</tbody>
</table>
### Math Maze

#### Questions and Answers for Math Maze Cards

| **Who has** the name of a polygon with all sides congruent and all angles congruent? | **I have** regular. |
| **Who has** the type of angle that measures exactly 90°? | **I have** right. |
| **Who has** the name of a six-sided polygon? | **I have** hexagon. |
| **Who has** the distance around a circle? | **I have** circumference. |
| **Who has** the name of a triangle with all sides measuring four inches? | **I have** equilateral. |
| **Who has** the name of a quadrilateral with four right angles? Its length and width are different. | **I have** rectangle. |
| **Who has** the name of a five-sided polygon? | **I have** pentagon. |
| **Who has** the name of a triangle with sides measuring four, five, and eight inches? | **I have** scalene. |
| **Who has** the distance around a polygon? | **I have** perimeter. |
| **Who has** the name of an angle measuring more than 90° but less than 180°? | **I have** obtuse. |
| **Who has** what you call two lines in the same plane that do not intersect? | **I have** parallel lines. |
| **Who has** a word describing the shape of a stop sign? | **I have** octagon. |
| **Who has** the name of a triangle with sides measuring six, six, and ten inches? | **I have** isosceles. |
| **Who has** the name of the line segment from one vertex of a rectangle to the opposite vertex? | **I have** diagonal. |
| **Who has** the type of angle that measures less than 90°? | **I have** acute. |
| **Who has** the name of a quadrilateral with exactly one pair of parallel sides? | **I have** trapezoid. |
| **Who has** what you call two lines that intersect to form a 90° angle? | **I have** perpendicular lines. |
| **Who has** a quadrilateral with four equal sides but without four equal angles? | **I have** rhombus. |
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the fraction equivalent to 0.25?</td>
<td>I have $\frac{1}{4}$.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{3}{10}$?</td>
<td>I have 0.3.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.15?</td>
<td>I have $\frac{3}{20}$.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{1}{8}$?</td>
<td>I have 0.125.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.375?</td>
<td>I have $\frac{3}{8}$.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{4}{7}$?</td>
<td>I have 0.8.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{4}{25}$?</td>
<td>I have 0.16.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{1}{7}$?</td>
<td>I have 0.2.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.17?</td>
<td>I have $\frac{1}{9}$.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{7}{20}$?</td>
<td>I have 0.35.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{3}{7}$?</td>
<td>I have 0.3.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.6?</td>
<td>I have $\frac{3}{5}$.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.4?</td>
<td>I have $\frac{2}{5}$.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.9?</td>
<td>I have $\frac{9}{10}$.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.55?</td>
<td>I have $\frac{11}{20}$.</td>
</tr>
<tr>
<td>Who has the fraction equivalent to 0.02?</td>
<td>I have $\frac{1}{50}$.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{1}{20}$?</td>
<td>I have 0.05.</td>
</tr>
<tr>
<td>Who has the decimal equivalent to $\frac{3}{7}$?</td>
<td>I have 0.6.</td>
</tr>
</tbody>
</table>
## Math Maze

### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the sum of $-3$ and $-8$?</td>
<td>$-11$.</td>
</tr>
<tr>
<td>Who has the sum of $9$ and $-4$?</td>
<td>$5$.</td>
</tr>
<tr>
<td>Who has the sum of $17$ and $14$?</td>
<td>$31$.</td>
</tr>
<tr>
<td>Who has the sum of $-15$ and $-4$?</td>
<td>$-19$.</td>
</tr>
<tr>
<td>Who has the sum of $5$ and $-5$?</td>
<td>$0$.</td>
</tr>
<tr>
<td>Who has the sum of $16$ and $-13$?</td>
<td>$3$.</td>
</tr>
<tr>
<td>Who has the sum of $-12$ and $-13$?</td>
<td>$-25$.</td>
</tr>
<tr>
<td>Who has the sum of $18$ and $-14$?</td>
<td>$4$.</td>
</tr>
<tr>
<td>Who has the sum of $8$ and $-2$?</td>
<td>$6$.</td>
</tr>
<tr>
<td>Who has the sum of $-10$ and $32$?</td>
<td>$22$.</td>
</tr>
<tr>
<td>Who has the sum of $-23$ and $-14$?</td>
<td>$-37$.</td>
</tr>
<tr>
<td>Who has the sum of $-12$ and $26$?</td>
<td>$14$.</td>
</tr>
<tr>
<td>Who has the sum of $-14$ and $-8$ and $-12$?</td>
<td>$-34$.</td>
</tr>
<tr>
<td>Who has the sum of $26$ and $-19$?</td>
<td>$7$.</td>
</tr>
<tr>
<td>Who has the sum of $-15$ and $24$?</td>
<td>$9$.</td>
</tr>
<tr>
<td>Who has the sum of $-19$ and $-12$?</td>
<td>$-31$.</td>
</tr>
<tr>
<td>Who has the sum of $5$ and $-5$ and $-12$?</td>
<td>$-12$.</td>
</tr>
</tbody>
</table>

## Week 8•Activity 36

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the greatest common factor of $12a$ and $15a$?</td>
<td>$3a$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $8a$ and $12a$?</td>
<td>$4a$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $3$ and $16$?</td>
<td>$48$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $10$ and $15$?</td>
<td>$30$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $8$ and $16$?</td>
<td>$8$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $11$ and $5$?</td>
<td>$55$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $5$ and $16$?</td>
<td>$80$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $35a$ and $7a$?</td>
<td>$7a$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $15a$ and $5a$?</td>
<td>$5a$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $9$ and $12$?</td>
<td>$36$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $7$ and $8$?</td>
<td>$56$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $30a$ and $24a$?</td>
<td>$6a$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $9$ and $6$?</td>
<td>$18$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $15$ and $9$?</td>
<td>$45$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $6a$ and $14a$?</td>
<td>$2a$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $10a$ and $27a$?</td>
<td>$a$.</td>
</tr>
<tr>
<td>Who has the greatest common factor of $16a$ and $32a$?</td>
<td>$16a$.</td>
</tr>
<tr>
<td>Who has the least common multiple of $8$ and $10$?</td>
<td>$40$.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the value of $3x + 1$, if $x = -4$?</td>
<td>I have -11.</td>
</tr>
<tr>
<td>Who has the value of $x - 12$, if $x = 5$?</td>
<td>I have -7.</td>
</tr>
<tr>
<td>Who has the value of $-20 ÷ x$, if $x = 4$?</td>
<td>I have -5.</td>
</tr>
<tr>
<td>Who has the value of $x^2$, if $x = -6$?</td>
<td>I have 36.</td>
</tr>
<tr>
<td>Who has the value of $4x - 3$, if $x = 8$?</td>
<td>I have 29.</td>
</tr>
<tr>
<td>Who has the value of $x + 5x$, if $x = 8$?</td>
<td>I have 48.</td>
</tr>
<tr>
<td>Who has the value of $8 + 3x$, if $x = -6$?</td>
<td>I have -10.</td>
</tr>
<tr>
<td>Who has the value of $7(x + 1)$, if $x = 6$?</td>
<td>I have 49.</td>
</tr>
<tr>
<td>Who has the value of $10 + 4x$, if $x = -8$?</td>
<td>I have -22.</td>
</tr>
<tr>
<td>Who has the value of $5x - 2$, if $x = -5$?</td>
<td>I have -27.</td>
</tr>
<tr>
<td>Who has the value of $x^2 - 3$, if $x = 6$?</td>
<td>I have 33.</td>
</tr>
<tr>
<td>Who has the value of $x ÷ -9$, if $x = -54$?</td>
<td>I have 6.</td>
</tr>
<tr>
<td>Who has the value of $24 - 2x$, if $x = -7$?</td>
<td>I have 38.</td>
</tr>
<tr>
<td>Who has the value of $-3(x + 2)$, if $x = 8$?</td>
<td>I have -30.</td>
</tr>
<tr>
<td>Who has the value of $x + x + x$, if $x = 7$?</td>
<td>I have 21.</td>
</tr>
<tr>
<td>Who has the value of $2x - 9$, if $x = 11$?</td>
<td>I have 13.</td>
</tr>
<tr>
<td>Who has the value of $x^2 + 4x$, if $x = 4$?</td>
<td>I have 32.</td>
</tr>
<tr>
<td>Who has the value of $16 + 2x$, if $x = -12$?</td>
<td>I have -8.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Who has the sum of $5\frac{2}{3}$ and $\frac{4}{5}$?</td>
<td>I have $6\frac{5}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $2\frac{1}{3}$ and $\frac{1}{6}$?</td>
<td>I have $2\frac{1}{2}$.</td>
</tr>
<tr>
<td>Who has the sum of $4\frac{1}{3}$ and $\frac{1}{3}$?</td>
<td>I have $4\frac{2}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{2}{3}$ and $\frac{2}{3}$?</td>
<td>I have $1\frac{1}{3}$.</td>
</tr>
<tr>
<td>Who has the sum of $3\frac{5}{6}$ and $\frac{1}{4}$?</td>
<td>I have $4\frac{1}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{1}{6}$ and $\frac{1}{6}$?</td>
<td>I have $\frac{1}{3}$.</td>
</tr>
<tr>
<td>Who has the sum of $2\frac{1}{5}$ and $\frac{1}{2}$?</td>
<td>I have $2\frac{2}{5}$.</td>
</tr>
<tr>
<td>Who has the sum of $1\frac{1}{4}$ and $1\frac{3}{4}$?</td>
<td>I have $2\frac{7}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{5}{6}$ and $\frac{1}{2}$?</td>
<td>I have $1\frac{7}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $1\frac{1}{3}$ and $\frac{2}{3}$?</td>
<td>I have $2$.</td>
</tr>
<tr>
<td>Who has the sum of $2\frac{2}{3}$ and $\frac{1}{3}$?</td>
<td>I have $3$.</td>
</tr>
<tr>
<td>Who has the sum of $2\frac{1}{4}$ and $2\frac{1}{4}$?</td>
<td>I have $4\frac{1}{2}$.</td>
</tr>
<tr>
<td>Who has the sum of $1\frac{3}{4}$ and $\frac{3}{4}$?</td>
<td>I have $1\frac{11}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{1}{4}$ and $\frac{1}{4}$ and $\frac{1}{4}$?</td>
<td>I have $\frac{3}{4}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{1}{6}$ and $\frac{1}{6}$ and $\frac{1}{6}$?</td>
<td>I have $\frac{1}{2}$.</td>
</tr>
<tr>
<td>Who has the sum of $3\frac{1}{2}$ and $\frac{2}{3}$?</td>
<td>I have $4\frac{1}{6}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{3}{6}$ and $\frac{3}{6}$?</td>
<td>I have $\frac{3}{12}$.</td>
</tr>
<tr>
<td>Who has the sum of $\frac{5}{6}$ and $\frac{3}{4}$ and $\frac{3}{4}$?</td>
<td>I have $2\frac{1}{4}$.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has $6 + (-2)$?</td>
<td>I have 8.</td>
</tr>
<tr>
<td>Who has $-7 + (-8)$?</td>
<td>I have 15.</td>
</tr>
<tr>
<td>Who has $4 - (-7)$?</td>
<td>I have 11.</td>
</tr>
<tr>
<td>Who has $3 + (-9 + 4)$?</td>
<td>I have 2.</td>
</tr>
<tr>
<td>Who has $-12 + (-7)$?</td>
<td>I have 5.</td>
</tr>
<tr>
<td>Who has $6 + (-2)$?</td>
<td>I have 4.</td>
</tr>
<tr>
<td>Who has $41 + (-7)$?</td>
<td>I have -3.</td>
</tr>
<tr>
<td>Who has $11 + (-12)$?</td>
<td>I have -1.</td>
</tr>
<tr>
<td>Who has $-7 + (-2)$?</td>
<td>I have 9.</td>
</tr>
<tr>
<td>Who has $14 + (-8)$?</td>
<td>I have 6.</td>
</tr>
<tr>
<td>Who has $11 + (-12)$?</td>
<td>I have 23.</td>
</tr>
<tr>
<td>Who has $-9 + (-5)$?</td>
<td>I have 14.</td>
</tr>
<tr>
<td>Who has $-15 + (-8)$?</td>
<td>I have 7.</td>
</tr>
<tr>
<td>Who has $5 + (-5)$?</td>
<td>I have 10.</td>
</tr>
<tr>
<td>Who has $14 + (-8)$?</td>
<td>I have 22.</td>
</tr>
<tr>
<td>Who has $5 + (-5)$?</td>
<td>I have 0.</td>
</tr>
<tr>
<td>Who has $-9 + (-13)$?</td>
<td>I have -4.</td>
</tr>
<tr>
<td>Who has $-11 + (-16)$?</td>
<td>I have -5.</td>
</tr>
</tbody>
</table>

### Week 12•Activity 56

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has $n$ if $3n = -42$?</td>
<td>I have -14.</td>
</tr>
<tr>
<td>Who has $n$ if $4n - 10 = -50$</td>
<td>I have -10.</td>
</tr>
<tr>
<td>Who has $n$ if $n - 13 = 17$</td>
<td>I have 30.</td>
</tr>
<tr>
<td>Who has $n$ if $n + 9 = -45$</td>
<td>I have 54.</td>
</tr>
<tr>
<td>Who has $n$ if $9n + 10 = 55$</td>
<td>I have 5.</td>
</tr>
<tr>
<td>Who has $n$ if $12 - n = 15$</td>
<td>I have 3.</td>
</tr>
<tr>
<td>Who has $n$ if $n + (-8) = 3$</td>
<td>I have -24.</td>
</tr>
<tr>
<td>Who has $n$ if $3n - 9 = 3$</td>
<td>I have 4.</td>
</tr>
<tr>
<td>Who has one value of $n$ if $n^2 = 81$?</td>
<td>I have 9.</td>
</tr>
<tr>
<td>Who has $n$ if $12n = -144$?</td>
<td>I have -12.</td>
</tr>
<tr>
<td>Who has $n$ if $54 = -2n$?</td>
<td>I have -27.</td>
</tr>
<tr>
<td>Who has $n$ if $10n + 35 = 145$?</td>
<td>I have 11.</td>
</tr>
<tr>
<td>Who has $n$ if $8n = -104$?</td>
<td>I have -13.</td>
</tr>
<tr>
<td>Who has $n$ if $2n + 3n = 40$?</td>
<td>I have 8.</td>
</tr>
<tr>
<td>Who has $n$ if $12 + n = -34$?</td>
<td>I have -46.</td>
</tr>
<tr>
<td>Who has $n$ if $\sqrt{n} = 10$?</td>
<td>I have 50.</td>
</tr>
<tr>
<td>Who has $n$ if $4n + 20 = 100$?</td>
<td>I have 20.</td>
</tr>
<tr>
<td>Who has $n$ if $19n = -19$?</td>
<td>I have 1.</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.25?</td>
<td>I have 25%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{3}{4}$?</td>
<td>I have 80%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 1.5?</td>
<td>I have 150%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.15?</td>
<td>I have 15%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{1}{5}$?</td>
<td>I have 20%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.01?</td>
<td>I have 1%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{1}{3}$?</td>
<td>I have $33\frac{1}{3}$%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{1}{2}$?</td>
<td>I have 75%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.48?</td>
<td>I have 48%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{1}{4}$?</td>
<td>I have $12\frac{1}{2}$%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{1}{2}$?</td>
<td>I have 50%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.375?</td>
<td>I have $37\frac{1}{2}$%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.05?</td>
<td>I have 5%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 0.66?</td>
<td>I have 66%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{2}{3}$?</td>
<td>I have $66\frac{2}{3}$%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to 1?</td>
<td>I have 100%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{7}{10}$?</td>
<td>I have 7%.</td>
</tr>
<tr>
<td>Who has the percent equivalent to $\frac{1}{10}$?</td>
<td>I have 10%.</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Who has the only even prime number?</td>
<td>I have 2.</td>
</tr>
<tr>
<td>Who has a prime number between 40 and 50? The product of its digits is 12.</td>
<td>I have 43.</td>
</tr>
<tr>
<td>Who has a composite number whose factors are one, seven, 13, and the number itself?</td>
<td>I have 91.</td>
</tr>
<tr>
<td>Who has the greatest two-digit prime number?</td>
<td>I have 97.</td>
</tr>
<tr>
<td>Who has the greatest two-digit multiple of three less than $2^4 \times 5$?</td>
<td>I have 78.</td>
</tr>
<tr>
<td>Who has the only counting number that is neither prime nor composite?</td>
<td>I have 1.</td>
</tr>
<tr>
<td>Who has the number whose prime factorization is $2^3 \times 3^2$?</td>
<td>I have 72.</td>
</tr>
<tr>
<td>Who has the greatest perfect square less than 100?</td>
<td>I have 81.</td>
</tr>
<tr>
<td>Who has a number whose factors are one, three, 17, and the number itself?</td>
<td>I have 51.</td>
</tr>
<tr>
<td>Who has the greatest one-digit perfect cube?</td>
<td>I have 8.</td>
</tr>
<tr>
<td>Who has the least number between 50 and 60 that is prime?</td>
<td>I have 53.</td>
</tr>
<tr>
<td>Who has the greatest two-digit multiple of eight?</td>
<td>I have 96.</td>
</tr>
<tr>
<td>Who has the least number that is divisible by both 12 and 9?</td>
<td>I have 36.</td>
</tr>
<tr>
<td>Who has the greatest two-digit perfect cube?</td>
<td>I have 64.</td>
</tr>
<tr>
<td>Who has the greatest prime factor of 92?</td>
<td>I have 23.</td>
</tr>
<tr>
<td>Who has the least odd prime number?</td>
<td>I have 3.</td>
</tr>
<tr>
<td>Who has a multiple of five? The sum of its digits is 11.</td>
<td>I have 65.</td>
</tr>
<tr>
<td>Who has the greatest prime number less than $2 \times 5^2$?</td>
<td>I have 47.</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Who has an expression equivalent to $8(x + 1)$?</td>
<td>I have $8x + 8$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $3(x - 7)$?</td>
<td>I have $3x - 21$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $3(x + 4)$?</td>
<td>I have $3x + 12$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $2(3 + 5x)$?</td>
<td>I have $10x + 6$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $-4(x + 7)$?</td>
<td>I have $-4x - 28$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $\frac{1}{2}(4x + 6)$?</td>
<td>I have $2x + 3$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $\frac{2}{3}(3x - 6)$?</td>
<td>I have $2x - 4$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $-2(3x + 4)$?</td>
<td>I have $-6x - 8$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $\frac{1}{3}(12 + 6x)$?</td>
<td>I have $2x + 4$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $-5(x - 1)$?</td>
<td>I have $-5x + 5$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $-2(6 + 4x)$?</td>
<td>I have $-8x - 12$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $\frac{1}{2}(8x - 4)$?</td>
<td>I have $4x - 2$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $2(3x + 4)$?</td>
<td>I have $6x + 8$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $5(x - 2)$?</td>
<td>I have $5x - 10$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $5(x + 3)$?</td>
<td>I have $5x + 15$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $4(2x + 1)$?</td>
<td>I have $8x + 4$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $\frac{1}{4}(4x + 4)$?</td>
<td>I have $x + 1$.</td>
</tr>
<tr>
<td>Who has an expression equivalent to $-3(2x + 3)$?</td>
<td>I have $-6x - 9$.</td>
</tr>
</tbody>
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Questions and Answers for Math Maze Cards

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<tr>
<td>Who has a value equivalent to $\frac{2}{3}$?</td>
<td>I have 120%.</td>
</tr>
<tr>
<td>Who has a value equivalent to 35%?</td>
<td>I have 0.35.</td>
</tr>
<tr>
<td>Who has a value equivalent to 0.6?</td>
<td>I have $\frac{3}{5}$.</td>
</tr>
<tr>
<td>Who has a value equivalent to 6%?</td>
<td>I have 0.06.</td>
</tr>
<tr>
<td>Who has a value equivalent to 12.5%?</td>
<td>I have $\frac{1}{8}$.</td>
</tr>
<tr>
<td>Who has a value equivalent to 1.25?</td>
<td>I have 125%.</td>
</tr>
<tr>
<td>Who has a value equivalent to $\frac{1}{3}$?</td>
<td>I have 33.3%.</td>
</tr>
<tr>
<td>Who has a value equivalent to 175%?</td>
<td>I have $\frac{7}{4}$.</td>
</tr>
<tr>
<td>Who has a value equivalent to 150%?</td>
<td>I have $\frac{3}{2}$.</td>
</tr>
<tr>
<td>Who has a value equivalent to $\frac{13}{20}$</td>
<td>I have 65%.</td>
</tr>
<tr>
<td>Who has a value equivalent to $2\frac{1}{2}$?</td>
<td>I have 250%.</td>
</tr>
<tr>
<td>Who has a value equivalent to 0.03?</td>
<td>I have $\frac{3}{100}$.</td>
</tr>
<tr>
<td>Who has a value equivalent to 0.55?</td>
<td>I have $\frac{11}{20}$.</td>
</tr>
<tr>
<td>Who has a value equivalent to $\frac{3}{4}$?</td>
<td>I have 0.375.</td>
</tr>
<tr>
<td>Who has a value equivalent to $\frac{7}{3}$?</td>
<td>I have 66\frac{2}{3}%.</td>
</tr>
<tr>
<td>Who has a value equivalent to 2?</td>
<td>I have 200%.</td>
</tr>
<tr>
<td>Who has a value equivalent to 5%?</td>
<td>I have 0.05.</td>
</tr>
<tr>
<td>Who has a value equivalent to 0.1?</td>
<td>I have $\frac{1}{10}$.</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>--------------------------------</td>
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</tr>
<tr>
<td>Who has the difference of -13 and -8?</td>
<td>I have -5.</td>
</tr>
<tr>
<td>Who has the difference of 6 and -1?</td>
<td>I have 7.</td>
</tr>
<tr>
<td>Who has the difference of 7 and 5?</td>
<td>I have 2.</td>
</tr>
<tr>
<td>Who has the difference of -5 and -4?</td>
<td>I have -1.</td>
</tr>
<tr>
<td>Who has the difference of 5 and -5?</td>
<td>I have 10.</td>
</tr>
<tr>
<td>Who has the difference of 6 and -3?</td>
<td>I have 9.</td>
</tr>
<tr>
<td>Who has the difference of -2 and -7?</td>
<td>I have 5.</td>
</tr>
<tr>
<td>Who has the difference of 7 and -5?</td>
<td>I have 12.</td>
</tr>
<tr>
<td>Who has the difference of -8 and 2?</td>
<td>I have -10.</td>
</tr>
<tr>
<td>Who has the difference of -10 and 2?</td>
<td>I have -12.</td>
</tr>
<tr>
<td>Who has the difference of -3 and -4?</td>
<td>I have 1.</td>
</tr>
<tr>
<td>Who has the difference of 5 and -6?</td>
<td>I have 11.</td>
</tr>
<tr>
<td>Who has the difference of -2 and 6?</td>
<td>I have -8.</td>
</tr>
<tr>
<td>Who has the difference of 5 and 7?</td>
<td>I have -2.</td>
</tr>
<tr>
<td>Who has the difference of 6 and -9?</td>
<td>I have 15.</td>
</tr>
<tr>
<td>Who has the difference of -2 and 4?</td>
<td>I have -6.</td>
</tr>
<tr>
<td>Who has the difference of 9 and -7?</td>
<td>I have 16.</td>
</tr>
<tr>
<td>Who has the difference of 3 and 12?</td>
<td>I have -9.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Who has</strong> the number of feet in six yards?</td>
<td>I have 18.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of inches in $1 \frac{1}{2}$ yards?</td>
<td>I have 54.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of years in three decades?</td>
<td>I have 30.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of cups in six quarts?</td>
<td>I have 24.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of pounds in $\frac{1}{10}$ ton?</td>
<td>I have 200.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of minutes in $1 \frac{3}{4}$ hours?</td>
<td>I have 105.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of seconds in $\frac{3}{4}$ minute?</td>
<td>I have 40.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of dimes in $3 \frac{1}{2}$ dollars?</td>
<td>I have 35.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of years in $\frac{1}{2}$ century?</td>
<td>I have 50.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of teaspoons in four tablespoons?</td>
<td>I have 12.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of minutes in $\frac{1}{10}$ of an hour?</td>
<td>I have 6.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of months in four years?</td>
<td>I have 48.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of ounces in $2 \frac{1}{4}$ pounds?</td>
<td>I have 36.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of grams in $\frac{1}{2}$ kilogram?</td>
<td>I have 500.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of weeks in $\frac{1}{4}$ of a year?</td>
<td>I have 39.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of quarters in seven dollars?</td>
<td>I have 28.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of hours in $\frac{3}{8}$ of a day?</td>
<td>I have 20.</td>
</tr>
<tr>
<td><strong>Who has</strong> the number of nickels in three dollars?</td>
<td>I have 60.</td>
</tr>
</tbody>
</table>
Questions and Answers for Math Maze Cards

| Who has the mean of this set of numbers: 1, 4, 10? | I have 5. |
| Who has the median of this set of numbers: 1, 4, 10? | I have 4. |
| Who has the mode of this set of numbers: 1, 4, 10? | I have no mode. |
| Who has the mean of this set of numbers: 3, 6, 14, 21? | I have 11. |
| Who has the mean of this set of numbers: 2, 8, 8? | I have 6. |
| Who has the mode of this set of numbers: 3, 6, 14, 14? | I have 14. |
| Who has the range of these numbers: 2, 7, 8, 9? | I have 7. |
| Who has the median of this set of numbers: 3, 6, 14, 21? | I have 10. |
| Who has the range of these numbers: 6, 9, 12, 15? | I have 9. |
| Who has the mean of this set of numbers: 12, 12, 14, 14? | I have 13. |
| Who has the median of this set of numbers: 1, 2, 33? | I have 2. |
| Who has the mean of this set of numbers: 14, 14, 20? | I have 16. |
| Who has the median of this set of numbers: 6, 15, 21, 22? | I have 18. |
| Who has the mode of this set of numbers: 2, 8, 8? | I have 8. |
| Who has the median of this set of numbers: 5, 15, 20? | I have 15. |
| Who has the range of these numbers: 15, 20, 40? | I have 25. |
| Who has the mode of this set of numbers: 3, 3, 14, 21? | I have 3. |
| Who has the mean of this set of numbers: 1, 2, 33? | I have 12. |
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<th>Question</th>
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<td>Who has the difference of $\frac{3}{4}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{1}{4}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{3}{4}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{5}{12}$.</td>
</tr>
<tr>
<td>Who has the difference of $1\frac{1}{2}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{2}{3}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{3}{8}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{3}{8}$.</td>
</tr>
<tr>
<td>Who has the difference of $1\frac{1}{4}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{3}{4}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{3}{4}$ and $\frac{1}{8}$?</td>
<td>I have $\frac{3}{8}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{1}{2}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{1}{2}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{3}{4}$ and $\frac{1}{3}$?</td>
<td>I have $\frac{7}{12}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{1}{2}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{1}{2}$.</td>
</tr>
<tr>
<td>Who has the difference of $1\frac{1}{4}$ and $\frac{1}{12}$?</td>
<td>I have $1\frac{5}{12}$.</td>
</tr>
<tr>
<td>Who has the difference of $1\frac{3}{8}$ and $\frac{1}{12}$?</td>
<td>I have $\frac{11}{12}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{1}{2}$ and $\frac{1}{8}$?</td>
<td>I have $\frac{5}{8}$.</td>
</tr>
<tr>
<td>Who has the difference of $1\frac{1}{8}$ and $\frac{3}{8}$?</td>
<td>I have $\frac{5}{8}$.</td>
</tr>
<tr>
<td>Who has the difference of $\frac{3}{4}$ and $\frac{3}{8}$?</td>
<td>I have $0$.</td>
</tr>
</tbody>
</table>
## Questions and Answers for Math Maze Cards

| Who has the name of a five-sided polygon? | I have pentagon. |
| Who has the distance across a circle through its center? | I have diameter. |
| Who has the name of a three-dimensional figure that has two circular bases and a curved lateral surface? | I have cylinder. |
| Who has the name of an eight-sided polygon? | I have octagon. |
| Who has the name of a rectangle with four congruent sides? | I have square. |
| Who has the name of a polygon with six sides? | I have hexagon. |
| Who has the name of the side in a right triangle opposite the right angle? | I have hypotenuse. |
| Who has the name of a four-sided polygon with no parallel sides, but two sets of congruent sides? | I have kite. |
| Who has the name of a rectangular prism whose faces are squares? | I have cube. |
| Who has the name of a three-dimensional figure whose base is a polygon and whose other faces are triangles that share a common vertex? | I have pyramid. |
| Who has the name of a four-sided polygon with exactly one pair of parallel sides? | I have trapezoid. |
| Who has the word that describes two figures that are the same size and shape? | I have congruent. |
| Who has the name of a three-dimensional figure that has a circular base, a vertex, and a curved lateral surface? | I have cone. |
| Who has the distance from a point on a circle to its center? | I have radius. |
| Who has the word that describes a figure that can fold in half on top of itself, making two congruent halves? | I have symmetric. |
| Who has the name of a three-sided polygon? | I have triangle. |
| Who has the name of any four-sided polygon? | I have quadrilateral. |
| Who has the name of a quadrilateral whose opposite sides are parallel? | I have parallelogram. |
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<td>Who has 10% of 80?</td>
<td>I have 8.</td>
</tr>
<tr>
<td>Who has 25% of 80?</td>
<td>I have 20.</td>
</tr>
<tr>
<td>Who has 50% of 66?</td>
<td>I have 33.</td>
</tr>
<tr>
<td>Who has 10% of 84?</td>
<td>I have 8.4.</td>
</tr>
<tr>
<td>Who has 33(\frac{1}{3})% of 96?</td>
<td>I have 32.</td>
</tr>
<tr>
<td>Who has 10% of 160?</td>
<td>I have 16.</td>
</tr>
<tr>
<td>Who has 5% of 150?</td>
<td>I have 7.5.</td>
</tr>
<tr>
<td>Who has 25% of 48?</td>
<td>I have 21.</td>
</tr>
<tr>
<td>Who has 5% of 66?</td>
<td>I have 3.3.</td>
</tr>
<tr>
<td>Who has 25% of 60?</td>
<td>I have 15.</td>
</tr>
<tr>
<td>Who has 5% of 60?</td>
<td>I have 3.</td>
</tr>
<tr>
<td>Who has 10% of 60?</td>
<td>I have 6.</td>
</tr>
<tr>
<td>Who has 50% of 168?</td>
<td>I have 84.</td>
</tr>
<tr>
<td>Who has 10% of 96?</td>
<td>I have 9.6.</td>
</tr>
<tr>
<td>Who has 33(\frac{1}{3})% of 66?</td>
<td>I have 22.</td>
</tr>
<tr>
<td>Who has 25% of 8.4?</td>
<td>I have 2.1.</td>
</tr>
<tr>
<td>Who has 50% of 8.4?</td>
<td>I have 4.2.</td>
</tr>
<tr>
<td>Who has 10% of 66?</td>
<td>I have 6.6.</td>
</tr>
</tbody>
</table>

### Week 23•Activity 111

<table>
<thead>
<tr>
<th>Question</th>
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<tbody>
<tr>
<td>Who has the sum of 8.1 and 4.5?</td>
<td>I have 12.6.</td>
</tr>
<tr>
<td>Who has the difference of 6.9 and 2?</td>
<td>I have 4.9.</td>
</tr>
<tr>
<td>Who has the quotient of 4.48 and 4?</td>
<td>I have 1.12.</td>
</tr>
<tr>
<td>Who has the difference of 0.8 and 0.25?</td>
<td>I have 0.55.</td>
</tr>
<tr>
<td>Who has the product of 0.6 and 0.4?</td>
<td>I have 0.24.</td>
</tr>
<tr>
<td>Who has the sum of 5.2 and 0.16?</td>
<td>I have 5.36.</td>
</tr>
<tr>
<td>Who has the product of 0.02 and 8?</td>
<td>I have 0.16.</td>
</tr>
<tr>
<td>Who has the difference of 8 and 3.2?</td>
<td>I have 4.8.</td>
</tr>
<tr>
<td>Who has the quotient of 0.18 and 0.03?</td>
<td>I have 6.</td>
</tr>
<tr>
<td>Who has the sum of 5.13 and 0.44?</td>
<td>I have 5.57.</td>
</tr>
<tr>
<td>Who has the sum of 1.38 and 2?</td>
<td>I have 3.38.</td>
</tr>
<tr>
<td>Who has the difference of 7.45 and 2?</td>
<td>I have 5.45.</td>
</tr>
<tr>
<td>Who has the product of 1.2 and 0.12?</td>
<td>I have 0.144.</td>
</tr>
<tr>
<td>Who has the product of 0.07 and 0.8?</td>
<td>I have 0.056.</td>
</tr>
<tr>
<td>Who has the difference of 0.6 and 0.04?</td>
<td>I have 0.56.</td>
</tr>
<tr>
<td>Who has the sum of 0.6 and 0.84?</td>
<td>I have 1.44.</td>
</tr>
<tr>
<td>Who has the quotient of 0.64 and 0.4?</td>
<td>I have 1.6.</td>
</tr>
<tr>
<td>Who has the product of 0.3 and 0.42?</td>
<td>I have 0.126.</td>
</tr>
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<td>I have $\frac{4}{9}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{3}{4}$ and $\frac{2}{5}$?</td>
<td>I have $\frac{3}{10}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{3}{8}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{3}{16}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{1}{2}$ and $\frac{1}{6}$?</td>
<td>I have $\frac{1}{12}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{1}{4}$ and $\frac{3}{5}$?</td>
<td>I have $\frac{3}{20}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{3}{5}$ and $\frac{1}{4}$?</td>
<td>I have $\frac{3}{20}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{1}{2}$ and $\frac{1}{5}$?</td>
<td>I have $\frac{1}{10}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{3}{4}$ and $\frac{1}{3}$?</td>
<td>I have $\frac{3}{12}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{3}{5}$ and $\frac{1}{2}$?</td>
<td>I have $\frac{3}{10}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{5}{6}$ and $\frac{3}{8}$?</td>
<td>I have $\frac{5}{16}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{1}{3}$ and $\frac{3}{8}$?</td>
<td>I have $\frac{1}{8}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{5}{6}$ and $\frac{3}{5}$?</td>
<td>I have $\frac{3}{10}$.</td>
</tr>
<tr>
<td>Who has the product of $\frac{3}{4}$ and $\frac{1}{3}$?</td>
<td>I have $\frac{1}{12}$.</td>
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<tr>
<td>Who has (\frac{2}{3}) of 360?</td>
<td>I have 240.</td>
</tr>
<tr>
<td>Who has (\frac{5}{6}) of 1500?</td>
<td>I have 900.</td>
</tr>
<tr>
<td>Who has (\frac{3}{8}) of 560?</td>
<td>I have 210.</td>
</tr>
<tr>
<td>Who has (\frac{2}{9}) of 810?</td>
<td>I have 180.</td>
</tr>
<tr>
<td>Who has (\frac{1}{5}) of 2000?</td>
<td>I have 400.</td>
</tr>
<tr>
<td>Who has (\frac{1}{4}) of 1800?</td>
<td>I have 450.</td>
</tr>
<tr>
<td>Who has (\frac{1}{6}) of 1020?</td>
<td>I have 170.</td>
</tr>
<tr>
<td>Who has (\frac{1}{3}) of 960?</td>
<td>I have 320.</td>
</tr>
<tr>
<td>Who has (\frac{3}{7}) of 630?</td>
<td>I have 270.</td>
</tr>
<tr>
<td>Who has (\frac{1}{5}) of 420?</td>
<td>I have 140.</td>
</tr>
<tr>
<td>Who has (\frac{2}{5}) of 550?</td>
<td>I have 220.</td>
</tr>
<tr>
<td>Who has (\frac{5}{8}) of 960?</td>
<td>I have 600.</td>
</tr>
<tr>
<td>Who has (\frac{1}{2}) of 1740?</td>
<td>I have 870.</td>
</tr>
<tr>
<td>Who has (\frac{2}{3}) of 720?</td>
<td>I have 630.</td>
</tr>
<tr>
<td>Who has (\frac{1}{6}) of 960?</td>
<td>I have 120.</td>
</tr>
<tr>
<td>Who has (\frac{2}{7}) of 350?</td>
<td>I have 100.</td>
</tr>
<tr>
<td>Who has (\frac{3}{4}) of 640?</td>
<td>I have 480.</td>
</tr>
<tr>
<td>Who has (\frac{3}{5}) of 350?</td>
<td>I have 280.</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Who has</strong> the area of a square with a seven-inch side?</td>
<td><strong>I have</strong> 49 square inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the length of a side of a rhombus with a perimeter of 64 inches?</td>
<td><strong>I have</strong> 16 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the area of my parallelogram? Its base is three inches long and its height is twice its base.</td>
<td><strong>I have</strong> 18 square inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the perimeter of my rectangle? It is seven inches long and two inches wide.</td>
<td><strong>I have</strong> 18 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the length of a side of a square with an area of 64 square inches?</td>
<td><strong>I have</strong> 8 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the length of each side of an equilateral triangle with a perimeter of 42 inches?</td>
<td><strong>I have</strong> 14 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the height of my rhombus? Its base is 12 inches long and its area is 84 square inches.</td>
<td><strong>I have</strong> 7 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the lengths of the congruent sides of my isosceles triangle? Its perimeter is 15 inches and its non-congruent side is three inches long.</td>
<td><strong>I have</strong> 6 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the area of a rectangle that is 21 inches long and ( \frac{1}{2} ) as wide?</td>
<td><strong>I have</strong> 63 square inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the perimeter of my triangle? Its shortest side is five inches long, the middle side is one inch longer than the shortest side, and the longest side is three inches longer than the shortest side.</td>
<td><strong>I have</strong> 19 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the area of my trapezoid? Its bases are eight and 16 inches long and its height is 10 inches.</td>
<td><strong>I have</strong> 120 square inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the height of a rhombus that has a base one foot long and an area of 108 square inches?</td>
<td><strong>I have</strong> 9 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the length of the third side of a triangle if its perimeter is one yard, one of the sides is a foot long, and another side is ( 1 \frac{1}{2} ) feet long?</td>
<td><strong>I have</strong> 10 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the length of the short side of a rectangle if the long side is only one inch longer than the short side and the area is 30 square inches?</td>
<td><strong>I have</strong> 5 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the perimeter of a rhombus with sides 1 foot 3 inches long?</td>
<td><strong>I have</strong> 60 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the base of a parallelogram if its height is four inches and its area is eight square inches?</td>
<td><strong>I have</strong> 2 inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the area of my trapezoid? Its height is five inches and its bases are three and 21 inches long.</td>
<td><strong>I have</strong> 60 square inches.</td>
</tr>
<tr>
<td><strong>Who has</strong> the perimeter of a square with an area of 81 square inches?</td>
<td><strong>I have</strong> 36 inches.</td>
</tr>
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<td>Who has 10% of $25.40?</td>
<td>I have $2.54.</td>
</tr>
<tr>
<td>Who has 50% of $8.60?</td>
<td>I have $4.30.</td>
</tr>
<tr>
<td>Who has 20% of $42.80?</td>
<td>I have $8.56.</td>
</tr>
<tr>
<td>Who has 33(\frac{1}{3})% of $45.30?</td>
<td>I have $15.10.</td>
</tr>
<tr>
<td>Who has 10% of $8.60?</td>
<td>I have $0.86.</td>
</tr>
<tr>
<td>Who has 33(\frac{1}{3})% of $15.45?</td>
<td>I have $5.15.</td>
</tr>
<tr>
<td>Who has 15% of $8.60?</td>
<td>I have $1.29.</td>
</tr>
<tr>
<td>Who has 33(\frac{1}{3})% of $9.60?</td>
<td>I have $3.20.</td>
</tr>
<tr>
<td>Who has 5% of $42.80?</td>
<td>I have $2.14.</td>
</tr>
<tr>
<td>Who has 10% of $42.80?</td>
<td>I have $4.28.</td>
</tr>
<tr>
<td>Who has 5% of $25.40?</td>
<td>I have $1.27.</td>
</tr>
<tr>
<td>Who has 25% of $8.60?</td>
<td>I have $2.15.</td>
</tr>
<tr>
<td>Who has 15% of $25.40?</td>
<td>I have $3.81.</td>
</tr>
<tr>
<td>Who has 5% of $8.60?</td>
<td>I have $0.43.</td>
</tr>
<tr>
<td>Who has 33(\frac{1}{3})% of $3.12?</td>
<td>I have $1.04.</td>
</tr>
<tr>
<td>Who has 15% of $42.80?</td>
<td>I have $6.42.</td>
</tr>
<tr>
<td>Who has 20% of $25.40?</td>
<td>I have $5.08.</td>
</tr>
<tr>
<td>Who has 20% of $8.60?</td>
<td>I have $1.72.</td>
</tr>
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<tr>
<td>Who has the square of $-3$?</td>
<td>I have 9.</td>
</tr>
<tr>
<td>Who has $\frac{1}{2}$ cubed?</td>
<td>I have $\frac{1}{8}$.</td>
</tr>
<tr>
<td>Who has the square of $-10$?</td>
<td>I have 100.</td>
</tr>
<tr>
<td>Who has the square of $-\frac{1}{3}$?</td>
<td>I have $\frac{1}{9}$.</td>
</tr>
<tr>
<td>Who has $\frac{1}{4}$ cubed?</td>
<td>I have $\frac{1}{64}$.</td>
</tr>
<tr>
<td>Who has 3 cubed?</td>
<td>I have 27.</td>
</tr>
<tr>
<td>Who has the square of $-9$?</td>
<td>I have 81.</td>
</tr>
<tr>
<td>Who has the fifth power of $-2$?</td>
<td>I have $-32$.</td>
</tr>
<tr>
<td>Who has $\frac{1}{3}$ cubed?</td>
<td>I have $\frac{1}{27}$.</td>
</tr>
<tr>
<td>Who has the fourth power of $-\frac{1}{10}$</td>
<td>I have $\frac{1}{10,000}$.</td>
</tr>
<tr>
<td>Who has $-4$ cubed?</td>
<td>I have $-64$.</td>
</tr>
<tr>
<td>Who has 8 squared?</td>
<td>I have 64.</td>
</tr>
<tr>
<td>Who has $\frac{1}{2}$ to the fourth power?</td>
<td>I have $\frac{1}{16}$.</td>
</tr>
<tr>
<td>Who has the cube of $-6$?</td>
<td>I have $-216$.</td>
</tr>
<tr>
<td>Who has 6 squared?</td>
<td>I have 36.</td>
</tr>
<tr>
<td>Who has the cube of $-\frac{1}{10}$?</td>
<td>I have $-\frac{1}{1000}$.</td>
</tr>
<tr>
<td>Who has the fourth power of $-3$?</td>
<td>I have 81.</td>
</tr>
<tr>
<td>Who has the fourth power of $-1$?</td>
<td>I have 1.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has $3 + 6 \times -8$?</td>
<td>I have 45.</td>
</tr>
<tr>
<td>Who has $-3 \times -4 + 8$?</td>
<td>I have 20.</td>
</tr>
<tr>
<td>Who has $9 \times 8 - 22$?</td>
<td>I have 36.</td>
</tr>
<tr>
<td>Who has $7 + 5 \times 9$?</td>
<td>I have 52.</td>
</tr>
<tr>
<td>Who has $3 \times 8 - 11$?</td>
<td>I have 35.</td>
</tr>
<tr>
<td>Who has $-9(2 - 5)$?</td>
<td>I have 27.</td>
</tr>
<tr>
<td>Who has $18 \div 9 \times 7$?</td>
<td>I have 14.</td>
</tr>
<tr>
<td>Who has $8 \times 6 - 22$?</td>
<td>I have 70.</td>
</tr>
<tr>
<td>Who has $-5 + 2 \times 7$?</td>
<td>I have -19.</td>
</tr>
<tr>
<td>Who has $9 \times 3 \times 2$?</td>
<td>I have 54.</td>
</tr>
<tr>
<td>Who has $(28 - 4) \times 3$?</td>
<td>I have 72.</td>
</tr>
<tr>
<td>Who has $2 + 6 \times 7$?</td>
<td>I have 44.</td>
</tr>
<tr>
<td>Who has $4 \times 8 + 6$?</td>
<td>I have 26.</td>
</tr>
<tr>
<td>Who has $-3 \times 5 - 6$?</td>
<td>I have -21.</td>
</tr>
<tr>
<td>Who has $4 \times 9 \div 3$?</td>
<td>I have 12.</td>
</tr>
<tr>
<td>Who has $8 + 6 \times -9$?</td>
<td>I have 62.</td>
</tr>
<tr>
<td>Who has $18 + 10 - 4$?</td>
<td>I have 24.</td>
</tr>
<tr>
<td>Who has $32 - 16 + 7$?</td>
<td>I have 23.</td>
</tr>
</tbody>
</table>

### Week 30 Activity 146

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the value of $5x - 4$, if $x = -9$?</td>
<td>I have -49.</td>
</tr>
<tr>
<td>Who has the value of $8 + 7x$, if $x = 3$?</td>
<td>I have 29.</td>
</tr>
<tr>
<td>Who has the value of $\frac{1}{2} + 15$, if $x = -12$?</td>
<td>I have 12.</td>
</tr>
<tr>
<td>Who has the value of $x^2 - 7$, if $x = -10$?</td>
<td>I have 93.</td>
</tr>
<tr>
<td>Who has the value of $5(x + 4)$, if $x = 7$?</td>
<td>I have 55.</td>
</tr>
<tr>
<td>Who has the value of $-8x + 13$, if $x = 3$?</td>
<td>I have -11.</td>
</tr>
<tr>
<td>Who has the value of $2x + 6x$, if $x = -5$?</td>
<td>I have -40.</td>
</tr>
<tr>
<td>Who has the value of $-6x - 12$, if $x = 8$?</td>
<td>I have -60.</td>
</tr>
<tr>
<td>Who has the value of $x^2 + 9$, if $x = 4$?</td>
<td>I have 25.</td>
</tr>
<tr>
<td>Who has the value of $37 - 3x$, if $x = -9$?</td>
<td>I have 64.</td>
</tr>
<tr>
<td>Who has the value of $\frac{1}{2} + 23$, if $x = -15$?</td>
<td>I have 28.</td>
</tr>
<tr>
<td>Who has the value of $7(x - 8)$, if $x = 15$?</td>
<td>I have 49.</td>
</tr>
<tr>
<td>Who has the value of $-24 + 5x$, if $x = 8$?</td>
<td>I have 16.</td>
</tr>
<tr>
<td>Who has the value of $-9x - 12$, if $x = 6$?</td>
<td>I have -66.</td>
</tr>
<tr>
<td>Who has the value of $3x + 8x$, if $x = -8$?</td>
<td>I have -88.</td>
</tr>
<tr>
<td>Who has the value of $10x - 25$, if $x = 7$?</td>
<td>I have 45.</td>
</tr>
<tr>
<td>Who has the value of $x + 4 + 18$, if $x = -12$?</td>
<td>I have 15.</td>
</tr>
<tr>
<td>Who has the value of $-4(x + 9)$ if $x = -6$?</td>
<td>I have -12.</td>
</tr>
</tbody>
</table>
Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has the result of a 50% increase of 60?</td>
<td>I have 90.</td>
</tr>
<tr>
<td>Who has the result of a 100% increase of 100?</td>
<td>I have 200.</td>
</tr>
<tr>
<td>Who has the result of a 10% increase of 400?</td>
<td>I have 440.</td>
</tr>
<tr>
<td>Who has the result of a 50% decrease of 300?</td>
<td>I have 150.</td>
</tr>
<tr>
<td>Who has the result of a 200% increase of 60?</td>
<td>I have 180.</td>
</tr>
<tr>
<td>Who has the result of a 25% decrease of 400?</td>
<td>I have 300.</td>
</tr>
<tr>
<td>Who has the result of a 50% decrease of 800?</td>
<td>I have 400.</td>
</tr>
<tr>
<td>Who has the result of a 10% increase of 500?</td>
<td>I have 550.</td>
</tr>
<tr>
<td>Who has the result of a 20% decrease of 600?</td>
<td>I have 480.</td>
</tr>
<tr>
<td>Who has the result of a 75% decrease of 440?</td>
<td>I have 110.</td>
</tr>
<tr>
<td>Who has the result of a 300% increase of 60?</td>
<td>I have 240.</td>
</tr>
<tr>
<td>Who has the result of a 150% increase of 100?</td>
<td>I have 250.</td>
</tr>
<tr>
<td>Who has the result of a 25% decrease of 500?</td>
<td>I have 375.</td>
</tr>
<tr>
<td>Who has the result of a 10% decrease of 350?</td>
<td>I have 315.</td>
</tr>
<tr>
<td>Who has the result of a 10% increase of 350?</td>
<td>I have 385.</td>
</tr>
<tr>
<td>Who has the result of a 200% increase of 120?</td>
<td>I have 360.</td>
</tr>
<tr>
<td>Who has the result of a 75% decrease of 400?</td>
<td>I have 100.</td>
</tr>
<tr>
<td>Who has the result of a 20% decrease of 200?</td>
<td>I have 160.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

| Who has the greater number, $2^1$ or $3^2$? | I have $3^2$. |
| Who has the greater number, $3 \times 10^1$ or $100,000$? | I have $100,000$. |
| Who has the greater number, $-5\frac{1}{2}$ or $-4\frac{1}{4}$? | I have $-4\frac{1}{4}$. |
| Who has the greater number, $0.048$ or $0.4$? | I have $0.4$. |
| Who has the greater number, $-\frac{7}{10}$ or $-0.68$? | I have $-0.68$. |
| Who has the greater number $-\frac{3}{4}$ or $-\frac{5}{3}$? | I have $-\frac{5}{3}$. |
| Who has the greater number, $20^2 \times 10$ or $5 \times 10^1$? | I have $5 \times 10^1$. |
| Who has the greater number, $100 \times 5.6$ or $1000 - 560$? | I have $100 \times 5.6$. |
| Who has the greater number, $-\frac{5}{6}$ or $-\frac{7}{8}$? | I have $-\frac{5}{6}$. |
| Who has the greater number, $(0.2)^3$ or $(\frac{1}{2})^3$? | I have $(\frac{1}{2})^3$. |
| Who has the greater number, $-\frac{1}{3}$ or $-0.35$? | I have $-\frac{1}{3}$. |
| Who has the greater number, $\frac{3}{8}$ of $-416$ or $50\%$ of $-416$? | I have $\frac{3}{8}$ of $-416$. |
| Who has the greater number, $68 - 34$ or $(-10)^2$? | I have $68 - 34$. |
| Who has the greater number, $\frac{3}{8}$ or $\frac{3}{6}$? | I have $\frac{3}{8}$. |
| Who has the greater number, $-2100 \div 70$ or $7 - 5 \times 8$? | I have $-2100 \div 70$. |
| Who has the greater number, $(-0.9)^2$ or $0.9$? | I have $0.9$. |
| Who has the greater number, $70 \times 8$ or $80 \times 6$? | I have $70 \times 8$. |
| Who has the greater number, $\frac{7}{10}$ of $150$ or $3 \times 33$? | I have $\frac{7}{10}$ of $150$. |
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who has $3^2 + 9^2 + 10$?</td>
<td>I have 100.</td>
</tr>
<tr>
<td>Who has $-100 + 7 \times 10$?</td>
<td>I have -30.</td>
</tr>
<tr>
<td>Who has $-6 + (-6)$?</td>
<td>I have 30.</td>
</tr>
<tr>
<td>Who has $3^3 + 3^2 + 3$?</td>
<td>I have 39.</td>
</tr>
<tr>
<td>Who has $-3 \times -4 \times -5$?</td>
<td>I have -60.</td>
</tr>
<tr>
<td>Who has $3^4 + 3^2$?</td>
<td>I have 3.</td>
</tr>
<tr>
<td>Who has $9 - 9$?</td>
<td>I have -72.</td>
</tr>
<tr>
<td>Who has $(-3)^3 - (-2)^3$?</td>
<td>I have 17.</td>
</tr>
<tr>
<td>Who has $3 \times 5^2 - 10$?</td>
<td>I have 65.</td>
</tr>
<tr>
<td>Who has $2^3 - 3^2$?</td>
<td>I have -19.</td>
</tr>
<tr>
<td>Who has $(4^2) \div -8$?</td>
<td>I have 2.</td>
</tr>
<tr>
<td>Who has $-2 \times 5^2 - 6$?</td>
<td>I have -56.</td>
</tr>
<tr>
<td>Who has $4^2 + 4$?</td>
<td>I have 32.</td>
</tr>
<tr>
<td>Who has $5^2 - 5 \times 2$?</td>
<td>I have 15.</td>
</tr>
<tr>
<td>Who has $-10 - (6^2 + 2^3)$?</td>
<td>I have -50.</td>
</tr>
<tr>
<td>Who has $2^2 \times 2^2 + 8^2$?</td>
<td>I have 80.</td>
</tr>
<tr>
<td>Who has $(10)^2 \div -10$?</td>
<td>I have -10.</td>
</tr>
<tr>
<td>Who has $-2(6^2 + 8^2)$?</td>
<td>I have -200.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Who has the next number in the pattern</th>
<th>I have</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2, -4, -6, -8, . . .</td>
<td>-10.</td>
</tr>
<tr>
<td>1, 4, 9, 16, . . .</td>
<td>25.</td>
</tr>
<tr>
<td>$-\frac{1}{2}$, $\frac{1}{4}$, $-\frac{1}{8}$, . . .</td>
<td>$-\frac{1}{32}$.</td>
</tr>
<tr>
<td>36, 24, 12, 0, . . .</td>
<td>-12.</td>
</tr>
<tr>
<td>6, 9, 12, 15, . . .</td>
<td>18.</td>
</tr>
<tr>
<td>1, -2, 4, -8, . . .</td>
<td>16.</td>
</tr>
<tr>
<td>5, 1, -3, -7, . . .</td>
<td>-11.</td>
</tr>
<tr>
<td>$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, . . .</td>
<td>$\frac{5}{6}$.</td>
</tr>
<tr>
<td>7, 8, 10, 13, 17, . . .</td>
<td>22.</td>
</tr>
<tr>
<td>2, 3, 5, 7, . . .</td>
<td>11.</td>
</tr>
<tr>
<td>8, 4, 2, 1, . . .</td>
<td>$\frac{1}{2}$.</td>
</tr>
<tr>
<td>6, 11, 21, 41, . . .</td>
<td>81.</td>
</tr>
<tr>
<td>-11, -7, -3, 1, . . .</td>
<td>5.</td>
</tr>
<tr>
<td>$\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$, . . .</td>
<td>1.</td>
</tr>
<tr>
<td>7, 1, -5, -11, . . .</td>
<td>-17.</td>
</tr>
<tr>
<td>-2, -6, -18, -54, . . .</td>
<td>-162.</td>
</tr>
<tr>
<td>1, $\frac{1}{2}$, $\frac{1}{4}$, . . .</td>
<td>$\frac{1}{64}$.</td>
</tr>
<tr>
<td>1, 3, 7, 15, . . .</td>
<td>31.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th><strong>Who has</strong></th>
<th><strong>I have</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>the circumference of a circle with a radius of 7.5 cm?</td>
<td>$15\pi$ cm.</td>
</tr>
<tr>
<td>the area of a triangle with base 44 cm and height 10 cm?</td>
<td>220 square cm.</td>
</tr>
<tr>
<td>the perimeter of a rhombus with sides 18 cm long?</td>
<td>72 cm.</td>
</tr>
<tr>
<td>the length of a side of my rectangle? Its area is 7.5 sq. cm and the other side is 2.5 cm long.</td>
<td>3 cm.</td>
</tr>
<tr>
<td>the perimeter of an equilateral triangle with sides 1(\frac{1}{2}) cm long?</td>
<td>5 cm.</td>
</tr>
<tr>
<td>the area of a circle with a diameter of 10 cm?</td>
<td>$25\pi$ square cm.</td>
</tr>
<tr>
<td>the perimeter of my parallelogram? The short side is seven cm and the long side is five cm longer than the short side.</td>
<td>38 cm.</td>
</tr>
<tr>
<td>the perimeter of my rectangle? It has an area of 20 square cm and a length of five cm.</td>
<td>18 cm.</td>
</tr>
<tr>
<td>the area of a triangle with a height of 12 cm and base of nine cm?</td>
<td>54 square cm.</td>
</tr>
<tr>
<td>the length of each side of an equilateral triangle with a perimeter of 96 cm?</td>
<td>32 cm.</td>
</tr>
<tr>
<td>the area of my parallelogram? It has a base 24 cm long and its height is (\frac{1}{2}) as long as its base.</td>
<td>144 square cm.</td>
</tr>
<tr>
<td>the area of my trapezoid? Its height and shorter base are eight cm and the longer base is twice the shorter base.</td>
<td>96 square cm.</td>
</tr>
<tr>
<td>the perimeter of a regular hexagon, if each side measures 10.5 cm?</td>
<td>63 cm.</td>
</tr>
<tr>
<td>the area of my rhombus? Its base is 10 cm long and its height is 25% shorter than its base.</td>
<td>75 square cm.</td>
</tr>
<tr>
<td>the perimeter of my isosceles triangle? The congruent legs are 15.5 cm long and the base is 10 cm long.</td>
<td>41 cm.</td>
</tr>
<tr>
<td>the area of a circle with a diameter of 14 cm?</td>
<td>$49\pi$ square cm.</td>
</tr>
<tr>
<td>the length of the base of my isosceles triangle? The perimeter is 20 cm and the congruent legs are eight cm long.</td>
<td>4 cm.</td>
</tr>
<tr>
<td>the diameter of a circle with a circumference of 45(\pi) cm?</td>
<td>45 cm.</td>
</tr>
</tbody>
</table>
### Questions and Answers for Math Maze Cards

<table>
<thead>
<tr>
<th>Who has</th>
<th>I have</th>
</tr>
</thead>
<tbody>
<tr>
<td>the volume of a rectangular prism with dimensions two inches by four inches by ten inches?</td>
<td>80 cubic inches.</td>
</tr>
<tr>
<td>the volume of my cylinder? Its base is six inches in diameter and its height is five inches.</td>
<td>$45\pi$ cubic inches.</td>
</tr>
<tr>
<td>the surface area of a cube with edges five inches long?</td>
<td>150 square inches.</td>
</tr>
<tr>
<td>the surface area of my cylinder? Its base has a radius of three inches and its height is five inches.</td>
<td>$48\pi$ square inches.</td>
</tr>
<tr>
<td>the length of an edge of a cube whose volume is 64 cubic inches?</td>
<td>4 inches.</td>
</tr>
<tr>
<td>the volume of my prism? Its square base has a six-inch edge and its height is three inches.</td>
<td>108 cubic inches.</td>
</tr>
<tr>
<td>the surface area of my cylinder? Its base has a radius of five inches and its height is three inches.</td>
<td>$80\pi$ square inches.</td>
</tr>
<tr>
<td>the length of an edge of a cube whose surface area is 54 square inches?</td>
<td>three inches.</td>
</tr>
<tr>
<td>the volume of a rectangular prism with dimensions three inches by five inches by four inches?</td>
<td>60 cubic inches.</td>
</tr>
<tr>
<td>the height of my cylinder? The area of its base is $15\pi$ square inches and its volume is $90\pi$ cubic inches.</td>
<td>six inches.</td>
</tr>
<tr>
<td>the volume of my cylinder? Its base has a radius of six inches and its height is four inches.</td>
<td>$144\pi$ cubic inches.</td>
</tr>
<tr>
<td>the length of an edge of a cube whose volume is 125 cubic inches?</td>
<td>five inches.</td>
</tr>
<tr>
<td>the surface area of a rectangular prism with dimensions four inches by four inches by ten inches?</td>
<td>192 square inches.</td>
</tr>
<tr>
<td>the height of my rectangular prism? The area of its base is 12 square inches and its volume is 96 cubic inches.</td>
<td>eight inches.</td>
</tr>
<tr>
<td>the volume of a rectangular prism with dimensions six inches by two inches by two inches?</td>
<td>24 cubic inches.</td>
</tr>
<tr>
<td>the surface area of a cube with edges 10 inches long?</td>
<td>600 square inches.</td>
</tr>
<tr>
<td>the area of the base of my cylinder? Its volume is $28\pi$ cubic inches and its height is seven inches.</td>
<td>$4\pi$ square inches.</td>
</tr>
<tr>
<td>the volume of my prism? The area of its base is 10 square inches and its height is 15 inches.</td>
<td>150 cubic inches.</td>
</tr>
</tbody>
</table>
Activity Correlation

<table>
<thead>
<tr>
<th>Concepts (and diagnostic item numbers)</th>
<th>Activity Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare and order decimals, fractions, integers (8, 9, 13, 15, 18, 21, 23, 25)</td>
<td>34, 61, 76, 116, 119, 156</td>
</tr>
<tr>
<td>Fractions, decimals, percents (8, 9, 12, 13, 15, 18, 23, 25)</td>
<td>26-28, 44, 59, 61, 76, 106, 129</td>
</tr>
<tr>
<td>Equivalent forms (8, 9, 11, 12, 13, 15, 20, 21, 23, 24, 25)</td>
<td>10, 26, 27, 54, 76</td>
</tr>
<tr>
<td>Estimate (6, 10, 11, 18, 19, 20, 24)</td>
<td>25, 35, 40, 97, 98, 111, 160, 174</td>
</tr>
<tr>
<td>Factors and multiples (1-7, 14, 16, 17, 22)</td>
<td>1, 14, 18, 36, 74</td>
</tr>
<tr>
<td>Prime factorization (1-7)</td>
<td>1</td>
</tr>
<tr>
<td>Powers and roots (1-9, 19-21, 24, 25)</td>
<td>1, 14, 39, 49, 136, 161</td>
</tr>
<tr>
<td>Prime and composite numbers (2, 3, 7, 8, 14, 17, 22)</td>
<td>18, 66</td>
</tr>
<tr>
<td>Scientific notation (6)</td>
<td>33, 175</td>
</tr>
<tr>
<td>Divisibility tests (22)</td>
<td>9, 70, 154</td>
</tr>
<tr>
<td>Vocabulary (1-9)</td>
<td>66</td>
</tr>
</tbody>
</table>

Grade 7
Activity Correlation

<table>
<thead>
<tr>
<th>Concepts (and diagnostic item numbers)</th>
<th>Activity Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integers (2, 6–11, 24, 25, 28)</td>
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Activity Correlation

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